

# MORAL HAZARD VERSUS LIQUIDITY AND THE OPTIMAL TIMING OF UNEMPLOYMENT BENEFITS\*

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## Abstract

We show that an unemployment insurance scheme in which unemployment benefits decrease over the unemployment spell allows to separately estimate the liquidity and moral hazard effects of unemployment insurance. We empirically estimate these effects using Spanish administrative data in a Regression Kink Design (RKD) that exploits two kinks in the schedule of unemployment benefits with respect to prior labor income. We derive a “sufficient statistics” formula for the optimal level of unemployment benefits that generalizes results by [Chetty \(2008\)](#) for the case in which unemployment benefits are allowed to vary over the unemployment spell. We find that during the first six months of the unemployment spell moral hazard effects dominate liquidity effects and that the benefits of unemployment insurance are low relative to the costs. On the other hand, after the initial six months, liquidity effects explain about three quarters of the change in hazard rates, raising the value of providing insurance in that period.

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# 1 Introduction

The main rationale in favor of unemployment insurance is that it allows unemployed workers to smooth their consumption while unemployed. At the same time, unemployment insurance distorts the relative price of leisure and consumption, and therefore reduces the marginal incentive to search for a new job. From a normative point of view, unemployment insurance needs to trade off the consumption-smoothing benefits with the moral hazard costs induced by the provision of insurance.

[Chetty \(2008\)](#) shows that only part of the response of job search intensity to unemployment insurance is due to moral hazard, and that the remainder is a “liquidity effect”: if financial markets are incomplete and unemployed workers are unable to borrow, then the unemployed will have a strong incentive to search for a job. If this is the case, then raising the level of unemployment benefits provides additional liquidity to unemployed workers and alleviates the incompleteness of financial markets. Agents receiving these higher unemployment benefits reduce their job search effort partly because their borrowing constraints become less pressing (the liquidity effect) and partly because the relative price of leisure and consumption changes (the moral hazard effect). As shown by [Chetty \(2008\)](#), the relative importance of these two effects determines the optimal level of unemployment insurance.

The analysis by [Chetty \(2008\)](#) considers only unemployment benefits that are constant during the unemployment spell. However, in many real-world unemployment insurance schemes benefits do not remain constant during the unemployment spell. This is the case in Spain, where benefits decrease after an initial six-month period. A vast theoretical literature (e.g., [Hopenhayn and Nicolini, 1997](#)) shows that the timing of benefits matters, and finds that time-varying unemployment benefits are usually optimal. In contrast, in more recent work, [Shimer and Werning \(2008\)](#) show that when workers can borrow and save, then economic theory implies that a constant or nearly constant scheme is optimal.

In this paper we empirically address the optimality of unemployment benefits that vary over time. We use economic theory and the institutional details of unemployment insurance in Spain to show that the evolution of liquidity and moral hazard effects over the unemployment spell are non-parametrically identified given appropriate data and as long as an invertibility condition (which is observable in the data) is satisfied. We then empirically estimate liquidity and moral hazard effects using a Regression Kink Design

(RKD) that exploits kinks in the schedule of unemployment benefits with respect to prior labor income. Armed with the resulting estimates, we calculate optimal unemployment insurance levels for Spain using a “sufficient statistics” formula that generalizes results by [Chetty \(2008\)](#) for the case in which unemployment benefits are allowed to vary over the unemployment spell.

This paper falls within the “sufficient statistics” framework, which provides a bridge between structural econometrics and reduced form estimations. Using this approach, [Chetty \(2006\)](#) generalizes a prior result by [Baily \(1978\)](#) and shows that the optimal level of unemployment insurance is completely determined by three high-level statistics: the elasticity of unemployment duration to unemployment benefits, the change in consumption upon unemployment, and the degree of relative risk aversion of a representative worker. In more recent work, [Chetty \(2008\)](#) shows how to decompose the effect of unemployment benefits on job search effort into a moral hazard and a liquidity component. As mentioned before, this is done for the special case of unemployment benefits that are constant during the unemployment spell. The moral hazard and liquidity effects identified by [Chetty \(2008\)](#) are later empirically estimated for the US by [Landais \(2015\)](#) using a Regression Kink Design, the same method we use in our estimation. Other work using a Regression Kink Design for a similar purpose, and using data for Austria, is carried out by [Card, Lee, Pei, and Weber \(2015\)](#).

The work closest to the objective of this paper is the working paper by [Kolsrud, Landais, Nilsson, and Spinnewijn \(2015\)](#), which studies the dynamic aspect of unemployment insurance and the optimal timing of unemployment benefits both theoretically and empirically, using administrative data for Sweden. However, that paper does not attempt to separate moral hazard and liquidity effects à la [Chetty \(2008\)](#). Rather, it identifies the dynamic welfare benefits of unemployment insurance using consumption data (which they calculate as a residual). From a theoretical standpoint, the working paper by [Kolsrud, Landais, Nilsson, and Spinnewijn \(2015\)](#) follows the line of [Chetty \(2006\)](#), who uses consumption data, rather than [Chetty \(2008\)](#), who uses labor market data exclusively. One of the drawbacks of using consumption data is that it is necessary to assume a functional form for the utility function, including the level of risk aversion, and to restrict the ways in which utility differs between employed and unemployed workers.<sup>1</sup>

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<sup>1</sup>Another issue that arises when using consumption data is that the theoretical model from which the sufficient statistics optimality result is derived is formulated for an individual agent whereas consumption expenditure is usually measured at the household level and difficult to assign to a particular household

The first theoretical contribution of our paper is to provide a novel identification result of moral hazard and liquidity effects of unemployment insurance which relies on variation of unemployment benefits during an unemployment spell. This differs from the identification of the effects in the models of [Chetty \(2008\)](#) and [Landaïs \(2015\)](#) that are tailored to the US, where unemployment benefits are flat during the unemployment spell. With flat benefits the response of hazard rates to unemployment benefits does not contain enough information to separate moral hazard and liquidity effects; [Chetty \(2008\)](#) resorts to the use of lump-sum severance payments to approximate the liquidity effect and calculates the moral hazard effect as a residual whereas [Landaïs \(2015\)](#) uses jumps in the length of the unemployment coverage periods as an approximation to changes in benefit levels in order to disentangle moral hazard and liquidity effects.

In contrast, when unemployment benefits vary over the unemployment spell, then it is possible to identify moral hazard and liquidity effects solely from hazard rates. Using the same model as [Lentz and Tranaes \(2005\)](#), [Chetty \(2008\)](#), and [Landaïs \(2015\)](#), and exploiting intratemporal and intertemporal first order conditions, we obtain relationships that connect moral hazard and liquidity effects across periods. This connection across periods implies that there are just two unknown objects that need to be estimated. The US, with its flat unemployment benefits provides [Chetty \(2008\)](#) and [Landaïs \(2015\)](#) with a single equation for the identification, which is not enough to solve for two unknowns. In unemployment insurance schemes such as the one in Spain, which has 6 months of high unemployment benefits followed by 18 months of lower unemployment benefits, the theory provides us with two equations which can be used to solve for the two unknowns, and the moral hazard and liquidity effects (in all periods) are exactly identified using data on hazard rates alone.

The second theoretical contribution of this paper is to obtain a “sufficient statistics” formula for the optimal level of unemployment benefits and their timing. This formula is related to that by [Kolsrud, Landaïs, Nilsson, and Spinnewijn \(2015\)](#) but, in comparison to theirs, our formula does not require the use of consumption data, as all relevant empirical quantities can be estimated using Spanish administrative Social Security data using quasi-experimental methods. Therefore, we avoid the need to estimate consumption as a residual and, as stressed by [Chetty \(2008\)](#), we do not need to assume any particular functional form for the utility function, or to make an assumption on the

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member.

degree of relative risk aversion.

The moral hazard and liquidity effects of unemployment insurance are identified from two estimates: the effect on the beginning-of-spell hazard rate of raising benefits in the first 6 months of the spell and the effect on this same hazard rate of raising benefits in the following 18 months of the spell. For Spain, [Bover, Arellano, and Bentolila \(2002\)](#) and, using different data, [Arellano, Bentolila, and Bover \(2004\)](#) estimate the effect of unemployment insurance on hazard rates but focusing on the effect of unemployment insurance as a whole whereas the estimates required by the theory are the marginal effects of raising the benefit level. [Rebollo-Sanz and Rodríguez-Planas \(2015\)](#) estimate the effect that a reduction in replacement rates in second (18-month) period has on unemployment duration. They find that the reduction in replacement rates has no effect on unemployment duration in that second part of the unemployment spell but that there is an impact on duration in the first 6 months of the unemployment spell, which they interpret as forward-looking behavior. This forward-looking behavior is also present in our approach given that our identification result relies on a reaction at the start of an unemployment spell in response to a change in benefits that lies in the future.<sup>2</sup>

For the estimation of our variables of interest, an ideal experimental setting would have benefits increase in each of the two sub-periods covered by unemployment insurance (the first 6 months and the subsequent 18 months) for a random sample of the population and identify the effect by comparing treated with untreated workers. In lieu of this experimental design, we use a Regression Kink Design (RKD). In Spain, unemployment benefits are tied to labor income over the 180 working days prior to the onset of unemployment but are capped above and below at an amount that is a multiple of an index called IPREM. These caps induce kinks in the relationship between income and benefits. Using these kinks, and the methodology put forward by [Card, Lee, Pei, and Weber \(2015\)](#) and [Nielsen, Sørensen, and Taber \(2010\)](#), and that is also used by [Landais \(2015\)](#), the two estimates of interest can be obtained using a sample of administrative Social Security data (*Muestra Continua de Vidas Laborales*, MCVL). In order to calibrate the level of optimal unemployment benefits, in addition to the impact on hazard rates, we also need estimates of the effect of unemployment benefits on unemployment duration and duration while covered by unemployment insurance. We use the same Regression Kink Design to obtain these estimates.

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<sup>2</sup>[Barceló and Villanueva \(2016\)](#) and [Campos and Reggio \(2015\)](#) present further evidence of forward-looking behavior by Spanish households.

Our estimates imply that the moral hazard effect is stronger than the liquidity effect for benefits paid in the first 6-month period and that this result is reversed for benefits paid in the following 18-month period. In our preferred specification, the response of the probability of finding a job to an increase in unemployment benefits in the first 6-month period is composed of 70% moral hazard effect and 30% liquidity effect. For the subsequent 18-months in which unemployment benefits are at a lower level, the moral hazard effect amounts to 25% and the liquidity effect to 75% of the impact on the hazard rate. This implies that the marginal benefit of providing additional unemployment insurance is larger in the second period than in the first. When these numbers are compared to the cost of providing additional insurance according to the sufficient statistics formula, our estimates imply that benefits are too high in the first period. For the second period, we cannot reject the null hypothesis that benefits are at the optimal level. According to these results the optimal schedule of unemployment benefits is flatter than the one that is used in practice.

The paper proceeds as follows. We present the model and derive our main identification result and the formula for optimal benefits in Section 2. In Section 3 we present our empirical strategy, describe the context and the data used for our estimations. In Section 4 we report our estimation results and apply our formula for optimal unemployment insurance for Spain. We conclude in Section 5.

## 2 Theory

In this section we present a dynamic model that generalizes that of [Chetty \(2008\)](#). Using this model, we prove that moral hazard and liquidity effects can be disentangled in an environment in which unemployment benefits are set at (at least) two different levels during a spell covered by unemployment insurance. In this case, moral hazard and liquidity effects are non-parametrically identified from the response of hazard rates at the start of an unemployment spell to changes in the level of unemployment benefits at different points in time. Moreover, the fiscal externalities that are needed to ascertain whether unemployment benefits are optimal in the resulting sufficient statistics formula are also non-parametrically identified, in this case from the response of durations to changes in the level of unemployment benefits at different points in time.

## 2.1 Environment

In the model time is discrete and indexed by  $t = 0, 1, \dots, T - 1$ . Each period a worker can be in one of two mutually exclusive states: either employed or unemployed. At  $t = 0$  the worker is initially unemployed. Starting at  $t = 0$ , each period  $t$  the worker transitions into employment with a probability  $s_t \in [0, 1)$ , which is endogenously chosen by the worker. Employment is an absorbing state, so that once employed the probability of transitioning back into unemployment is zero.

## 2.2 Duration-dependent unemployment insurance

Unemployment benefits take two values.<sup>3</sup> For the first  $B_1$  periods in which the worker is unemployed, unemployment benefits are  $\bar{b}_1 > 0$ . For the next  $B_2$  periods, unemployment benefits are  $\bar{b}_2 > 0$ , and they revert to zero afterwards. Therefore, the total number of periods covered by unemployment insurance payments is  $B \equiv B_1 + B_2$  and the stream of duration-dependent unemployment benefits is captured by a  $T$ -dimensional vector:

$$\mathbf{b} = (\underbrace{\bar{b}_1, \dots, \bar{b}_1}_{0, \dots, B_1 - 1}, \underbrace{\bar{b}_2, \dots, \bar{b}_2}_{B_1, \dots, B_1 + B_2 - 1}, 0, \dots). \quad (1)$$

As usual, we define the survival rate in unemployment at date  $t$  as

$$S_t \equiv \prod_{i=0}^t (1 - s_i), \quad t \geq 0. \quad (2)$$

Using the definition of the survival rate, the expected duration of unemployment is given by

$$D \equiv \sum_{t=0}^{T-1} S_t. \quad (3)$$

To compute the expected cost of providing unemployment insurance, the expected durations during which the worker is entitled to benefit levels  $\bar{b}_1$  and  $\bar{b}_2$  are necessary. They are, respectively,

$$D_1 \equiv \sum_{t=0}^{B_1 - 1} S_t \quad (4)$$

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<sup>3</sup>The model can be easily extended to more than two values.

and

$$D_2 \equiv \sum_{t=B_1}^{B_1+B_2-1} S_t \quad (5)$$

With these definitions, the expected cost of providing unemployment benefits can be written as

$$\begin{aligned} C(\mathbf{b}) &= \bar{b}_1 \sum_{t=0}^{B_1-1} S_t + \bar{b}_2 \sum_{t=B_1}^{B_1+B_2-1} S_t \\ &= \bar{b}_1 D_1 + \bar{b}_2 D_2. \end{aligned} \quad (6)$$

### 2.3 The dynamic optimization problem of a worker

Agents take as given the sequence of wage rates  $w_t$ , unemployment benefits  $b_t$ , and taxes used to finance these unemployment benefits  $\tau_t$ . They also take as given any non-labor income  $a_t$ . Future periods are discounted at the discount rate  $\beta \in [0, 1]$ . Consumption in the employed state generates utility according to a function  $v(c)$  with  $v'(c) > 0$ ,  $v''(c) < 0$  for all  $c$ . Consumption in the unemployed state generates utility according to a function  $u(c)$ , with  $u'(c) > 0$ ,  $u''(c) < 0$  for all  $c$ . The utility of consumption in the employed and unemployed state may differ, for example, because of complementarities with leisure, psychological costs related to unemployment, etc. This is captured by using different functions  $u$  and  $v$  in both states.<sup>4</sup>

An agent who is out of work at the beginning of period  $t$  enters that period with assets  $A_t$  and chooses a probability  $s_t$  of transitioning into the employed state with an associated disutility  $\psi(s_t)$ . The value of being a job searcher is

$$J_t(A_t) = \max_{s_t} s_t V_t(A_t) + (1 - s_t) U_t(A_t) - \psi(s_t). \quad (7)$$

The function  $\psi : [0, 1] \rightarrow \mathbb{R}$  captures the disutility associated with the search effort for the chosen probability  $s$ . We assume that  $\psi'(s) > 0$  and  $\psi''(s) < 0$  for all  $s \in [0, 1]$ . The functions  $V_t(A_t)$  and  $U_t(A_t)$  are, respectively, the value functions of being employed and unemployed.

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<sup>4</sup>In comparison to our approach, studies that use consumption data to measure the benefits of unemployment insurance require the functions  $u$  and  $v$  to be the same or, at least, to be related in a known way.



The value function for an employed worker is

$$V_t(A_t) = \max_{A_{t+1} \in [L, A_t + a_t + w_t - \tau_t]} v(A_t - A_{t+1} + a_t + w_t - \tau_t) + \beta V_t(A_{t+1}). \quad (8)$$

The agent chooses next period's assets  $A_{t+1}$  and is constrained by a borrowing constraint in the form of a lower bound for assets  $L \leq 0$ . Because the employed state is absorbing, an employed agent faces no uncertainty.

The value function for an unemployed agent is

$$U_t(A_t) = \max_{A_{t+1} \in [L, A_t + a_t + b_t]} u(A_t - A_{t+1} + a_t + b_t) + \beta J_{t+1}(A_{t+1}). \quad (9)$$

The agent faces the same lower bound for assets as in the employed state. In contrast to what happens in the employed state, because  $J_{t+1}(A_{t+1})$  depends on the realization of future transitions into employment, unemployed agents do face uncertainty.<sup>5</sup>

Our model is identical to the model of [Chetty \(2008\)](#), with the exception that we allow  $b_t$  to change over the duration of a unemployment spell and also that we allow for  $\beta < 1$  to show that the results do not depend on the lack of discounting. In obtaining our results we will make use of two assumptions that are commonly made (e.g., [Chetty, 2008](#); [Landais, 2015](#)) and that we state up front.

**Assumption 1** *The borrowing constraint does not bind at any date  $t$  for  $0 \leq t \leq B - 1$ .*

This assumption allows us to use the agent's Euler equations to derive the optimal response of job search effort to a change in unemployment benefits in periods that lie in the future.<sup>6</sup>

**Assumption 2** *Consumption in the employed state at any date  $t$  does not depend on the length of the prior unemployment spell, that is,  $c_t^e$  is solely a function of  $t$  and does not depend on the history of realizations of the employment random variable.*

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<sup>5</sup>The problem for unemployed agents might therefore not be concave. [Lentz and Tranaes \(2005\)](#) show that this can be solved by introducing wealth lotteries. However, as shown by [Lentz and Tranaes \(2005\)](#) and [Chetty \(2008\)](#) in their simulations, nonconcavity does not arise in practice for sensible parameter values. We therefore follow the literature and assume that  $U_t$  is globally concave.

<sup>6</sup>A subtle point to be noted is that, although this assumption precludes the case in which the borrowing constraint is actually reached, the agent is aware of the constraint and acts considering that it may be reached. In fact, the agent by acting optimally reduces the likelihood that the constraint is ever reached, making the assumption more palatable.

This assumption should be taken as an approximation. If the length of an unemployment spell does not significantly alter lifetime wealth, then consumption once reemployed will not be unduly influenced by it. Formally, consumption in the employed state is a function of the full history  $h_t$ , which collects all the realizations of the random variable that governs transitions from the unemployed state to the employed state prior to and including date  $t$ . A proper notation for consumption in the employed state, one that takes into account this history-dependence, is therefore  $c^e(h_t)$ . Under Assumption 2, for any  $t \geq 1$ , we can write  $E[c_t^e] = \frac{1}{\sum_{j=0}^t \prod_{i=0}^{j-1} (1-s_i) s_j} \sum_{j=0}^t \prod_{i=0}^{j-1} (1-s_i) s_j c^e(h_t) = c_t^e$ . That is, consumption in the employed state depends only on the date  $t$ . This assumption is not used in the positive result in Proposition 1 below but will be useful to derive the normative result in Proposition 2.

## 2.4 Positive analysis

In the problem of a job searcher in (7), at an interior solution, the optimal search effort at date  $t$  satisfies

$$\psi'(s_t) = V_t(A_t) - U_t(A_t). \quad (10)$$

Intuitively, an unemployed worker sets  $s_t$  so as to equate  $V_t(A_t) - U_t(A_t)$ , the marginal benefit of obtaining a job, to the marginal cost of choosing search effort  $s_t$ , which is given by  $\psi'(s_t) > 0$ . This intra-temporal first order condition is behind the decomposition into a liquidity and moral hazard effect by Chetty (2008). We present this decomposition in its most general way in Lemma 1.

**Lemma 1** *For any date  $t$ , the effect on job search behavior at date  $t$  of increasing unemployment benefits  $j \geq 0$  periods in the future can be decomposed into a liquidity effect and a moral-hazard effect:*

$$\frac{\partial s_t}{\partial b_{t+j}} = \frac{\partial s_t}{\partial a_{t+j}} - \frac{\partial s_t}{\partial w_{t+j}}, \quad j \geq 0 \quad (11)$$

This result is intuitive from a finance perspective. Consider the question of how an agent responds to receiving an extra dollar  $j$  periods in the future when this dollar can be paid through instruments that pay off in different states of the world. Non-labor income  $a_{t+j}$  pays off in all possible states of the world that can occur at date  $t + j$ .

On the other hand, unemployment benefits  $b_{t+j}$  pay off only if the unemployed state is realized whereas the wage  $w_{t+j}$  pays off only if the employed state is realized at date  $t + j$ . Because an agent must be either employed or unemployed, increasing  $b_{t+j}$  by one dollar is equivalent to increasing  $a_{t+j}$  by one dollar while simultaneously reducing  $w_{t+j}$  by one dollar. Therefore, the effect of these two alternatives on  $s_t$  (or on any other endogenous variable) must be the same.

The economic intuition for Lemma 1 is that an increase in the unemployment benefit level in any period, present or future, lowers search intensity through two channels. The first channel,  $\frac{\partial s_t}{\partial a_{t+j}} \leq 0$ , is the liquidity effect, allowing the agent to maintain a higher level of consumption while unemployed and therefore reducing the urgency of finding a new job. The second channel,  $-\frac{\partial s_t}{\partial w_{t+j}} < 0$ , is the moral hazard effect; a higher benefit reduces the incentive to search through a change in the relative price of consumption and leisure.

Through the use of the intertemporal first order conditions, it is possible to relate the liquidity and moral hazard effects that lie  $j$  periods into the future with those of the current period. This result, stated in Lemma 2, allows us to greatly reduce the number of unobservable variables that need to be identified from the data.

**Lemma 2** *For any date  $t$ , and any  $j \geq 1$  such that the borrowing constraint does not bind in any period between  $t$  and  $t + j$ ,*

$$\frac{\partial s_t}{\partial a_{t+j}} = \frac{\partial s_t}{\partial a_t} \quad (12)$$

and

$$\frac{\partial s_t}{\partial w_{t+j}} = \frac{\partial s_t}{\partial w_t} \prod_{i=1}^j (1 - s_{t+i}). \quad (13)$$

This Lemma states that, if the borrowing constraint does not bind, and Euler equations hold with equality, then the effect on current job search of raising non-labor income  $a$  now or  $j$  periods in the future is the same. The agent's ability to smooth consumption intertemporally implies that the optimal choices made by the agent cannot be improved by reallocating resources purely across periods. Therefore, the effect on current search behavior does not depend on the timing of the payment.<sup>7</sup> Mathematically, the present

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<sup>7</sup>The fact that, because  $\beta < 1$ , the agent prefers earlier rather than later payments does not contradict the result: although utility depends on the timing of a payment, optimal search effort does not.

value of an extra dollar delivered through  $a$  in any future period raises the expressions  $V_t(A_t)$  and  $U_t(A_t)$  in (10) by exactly the same amount. Because current search effort depends on the difference of these two present values, the timing of  $a$  does not matter for the optimal search decision.

In the case of the moral hazard effect, however, the timing does matter. The thought experiment of paying an extra dollar through the wage implies that the extra amount of money is paid only in the states of the world in which the agent is employed. Therefore, the receipt of this dollar earlier or later is not an issue of purely intertemporal reallocation. Raising the wage in period  $t + j$  distorts the consumption-leisure choice only to the extent that the agent expects to still be unemployed in period  $t + j$ . The probability of still being unemployed drops with the horizon  $j$  because the agent has had more chances to reach the employed state. This results in that the moral hazard effect becomes weaker for periods that lie further in the future.

By substituting the expressions of Lemma 2 into the result of Lemma 1 the following Corollary is immediate.

**Corollary 1** *For any date  $t$ , and any  $j \geq 1$  such that the borrowing constraint does not bind in any period between  $t$  and  $t + j$ , the decomposition in Lemma 1 can be expressed as:*

$$\frac{\partial s_t}{\partial b_{t+j}} = \frac{\partial s_t}{\partial a_t} - \frac{\partial s_t}{\partial w_t} \prod_{i=1}^j (1 - s_{t+i}), \quad j \geq 1. \quad (14)$$

*In particular, by specializing to the case  $t = 0$ , if the borrowing constraint does not bind at date  $j$  or before, then:*

$$\frac{\partial s_0}{\partial b_j} = \frac{\partial s_0}{\partial a_0} - \frac{\partial s_0}{\partial w_0} \prod_{i=1}^j (1 - s_i), \quad j \geq 1. \quad (15)$$

The result in (15) allows to express the liquidity and moral hazard component of raising any current or future benefit as a function of the time-zero liquidity and moral hazard components. Because the liquidity and moral hazard effects are not directly observable, they must be inferred from relationships in the data. However, thanks to the connection of the liquidity and moral hazard effects across periods implied by Euler equations, we reduced the number of unknown variables, and therefore also the requirement put on the type of variation that needs to be observed in the data for identification.

The term  $\frac{\partial s_0}{\partial b_j}$  on the left hand side in (15) is not in general directly observable because benefits are constant over certain periods. Benefits in a certain period cannot change in isolation: they move in tandem over periods of length  $B_1$  and  $B_2$ . For example, if  $B_1$  lasts for more than one date, then the experiment of changing benefits just at the first date ( $j = 0$ ) will never be observed in practice, so that there will be no empirical counterpart to  $\frac{\partial s_0}{\partial b_0}$  but, given an appropriate empirical setting, it will be possible to infer the effect of a change at all dates belonging to the period of length  $B_1$ , i.e., the value of  $\sum_{t=0}^{B_1-1} \frac{\partial s_0}{\partial b_0}$ , from the data.

To take into account the constancy of benefits over the periods of length  $B_1$  and  $B_2$  we follow Chetty (2008) and define for any variable  $x \in \{a, b, w\}$  the notation:

$$\frac{\partial s_0}{\partial x} \Big|_{B_1} \equiv \sum_{t=0}^{B_1-1} \frac{\partial s_0}{\partial x_t} \quad \text{and} \quad \frac{\partial s_0}{\partial x} \Big|_{B_2} \equiv \sum_{t=B_1}^{B_1+B_2-1} \frac{\partial s_0}{\partial x_t} \quad (16)$$

Using this notation, we return to the original decomposition of Lemma 1 and write it taking into account the constancy of benefits over periods  $B_1$  and  $B_2$  to obtain aggregated expressions of the liquidity and moral hazard effects, similarly to what Chetty (2008) does for just one benefit level. Starting from the expression in (11), we set  $t = 0$  and, respectively,  $j = 0, \dots, B_1 - 1$  and  $j = B_1, \dots, B_1 + B_2 - 1$ . This results in

$$\frac{\partial s_0}{\partial \bar{b}_1} \equiv \frac{\partial s_0}{\partial b} \Big|_{B_1} = \underbrace{\frac{\partial s_0}{\partial a} \Big|_{B_1}}_{LIQ_1} - \underbrace{\frac{\partial s_0}{\partial w} \Big|_{B_1}}_{MH_1} \quad (17)$$

and

$$\frac{\partial s_0}{\partial \bar{b}_2} \equiv \frac{\partial s_0}{\partial b} \Big|_{B_2} = \underbrace{\frac{\partial s_0}{\partial a} \Big|_{B_2}}_{LIQ_2} - \underbrace{\frac{\partial s_0}{\partial w} \Big|_{B_2}}_{MH_2} \quad (18)$$

In the first of these equations, the expression on the left hand side ( $\frac{\partial s_0}{\partial \bar{b}_1}$ ) is the effect on search effort of varying  $\bar{b}_1$ , the level of unemployment benefits covering the whole period of length  $B_1$ . On the right hand side,  $LIQ_1 = \frac{\partial s_0}{\partial a} \Big|_{B_1}$  is the total liquidity effect associated to the change in  $\bar{b}_1$  and  $MH_1 = \frac{\partial s_0}{\partial w} \Big|_{B_1}$  is the total moral hazard effect associated to the change in  $\bar{b}_1$ . Likewise, in the second equation,  $LIQ_2$  and  $MH_2$  are liquidity and moral hazard effects of changing  $\bar{b}_2$ , the level of unemployment benefits covering the period of length  $B_2$ . Notice that in both equations the effect is on initial

search effort  $s_0$ . Because agents are forward-looking, they will optimally adjust  $s_0$  in response to changes in periods that lie in the future.

As we show later in our empirical application, it is possible to estimate the effect of benefit levels on search effort on the left hand side of (17) and (18) for Spain using a Regression Kink Design. Given these values, the following Proposition states that liquidity and moral hazard effects are non-parametrically identified provided an invertibility condition holds.

**Proposition 1** *Under Assumption 1, whenever  $\frac{D_1}{B_1} \neq \frac{D_2}{B_2}$ , the dynamic liquidity and moral hazard effects are non-parametrically identified from UI entitlement periods  $B_1, B_2$ , durations  $D_1, D_2$ , and  $\frac{\partial s_0}{\partial b_1}$  and  $\frac{\partial s_0}{\partial b_2}$ . The expressions for the liquidity and moral hazard effects are given by:*

$$\begin{aligned}
 LIQ_1 &= \left. \frac{\partial s_0}{\partial a} \right|_{B_1} = \frac{B_1}{B_2 D_1 - B_1 D_2} \left( D_1 \frac{\partial s_0}{\partial \bar{b}_2} - D_2 \frac{\partial s_0}{\partial \bar{b}_1} \right) \\
 MH_1 &= \left. \frac{\partial s_0}{\partial w} \right|_{B_1} = \frac{D_1}{B_2 D_1 - B_1 D_2} \left( B_1 \frac{\partial s_0}{\partial \bar{b}_2} - B_2 \frac{\partial s_0}{\partial \bar{b}_1} \right) \\
 LIQ_2 &= \left. \frac{\partial s_0}{\partial a} \right|_{B_2} = \frac{B_2}{B_2 D_1 - B_1 D_2} \left( D_1 \frac{\partial s_0}{\partial \bar{b}_2} - D_2 \frac{\partial s_0}{\partial \bar{b}_1} \right) \\
 MH_2 &= \left. \frac{\partial s_0}{\partial w} \right|_{B_2} = \frac{D_2}{B_2 D_1 - B_1 D_2} \left( B_1 \frac{\partial s_0}{\partial \bar{b}_2} - B_2 \frac{\partial s_0}{\partial \bar{b}_1} \right)
 \end{aligned} \tag{19}$$

This proposition provides our main theoretical result that we will use to empirically disentangle liquidity and moral hazard effects. Recall that expression (17) states that the effect of raising  $\bar{b}_1$  on search effort  $s_0$  can be decomposed into a liquidity effect  $LIQ_1$  and a moral hazard effect  $MH_1$  and that expression (18) does the same for  $LIQ_2$  and  $MH_2$ . However, liquidity and moral hazard effects are essentially unobservable. Proposition 1 proves that under Assumption 1 the unknown liquidity and moral hazard effects are identified from  $\frac{\partial s_0}{\partial \bar{b}_1}$  and  $\frac{\partial s_0}{\partial \bar{b}_2}$  and observable labor market variables.

The intuitive reason why the liquidity and moral hazard effect can be disentangled is that, for periods that lie further in the future, the moral hazard effect wears off more rapidly than the liquidity effect. This implies that the effect of raising benefits at the start of the unemployment spell will have a larger ratio of moral hazard to liquidity effect than raising benefits later in the unemployment spell. In fact, as can be seen from

the expressions in (19), over any period  $k$  during which benefits are constant at the level  $\bar{b}_k$ , the ratio of moral hazard effect to the liquidity effect is given by  $\frac{MH_k}{LIQ_k} = \frac{D_k}{B_k}\gamma$ , where  $\gamma$  is a constant that does not depend on  $k$ . Therefore, given any two periods for which  $\frac{1}{\gamma}\frac{MH_k}{LIQ_k} = \frac{D_k}{B_k} \neq \frac{D_{k'}}{B_{k'}} = \frac{1}{\gamma}\frac{MH_{k'}}{LIQ_{k'}}$ , the liquidity and moral hazard effects in periods  $k$  and  $k'$  can be identified.

The invertibility condition  $\frac{D_k}{B_k} \neq \frac{D_{k'}}{B_{k'}}$  needed to identify the effects separately will always be satisfied in practice because survival rates are non-increasing. By construction, non-increasing survival rates imply that if  $k$  and  $k'$  refer to two consecutive periods, then  $\frac{D_k}{B_k} \geq \frac{D_{k'}}{B_{k'}}$ . The inequality is strict except in the pathological case in which the survival rate stays constant over both periods, which would imply that transitioning out of unemployment is a zero probability event during the whole period covered by unemployment benefits at the levels  $\bar{b}_k$  and  $\bar{b}_{k'}$ .<sup>8</sup>

## 2.5 Normative analysis

We now turn to the normative aspects of the model. The dynamic liquidity and moral hazard effects identified in the previous section are informative of how changing unemployment benefit levels affects welfare of the representative worker. Coupled with information on the costs of providing unemployment insurance, the liquidity and moral hazard effects determine optimal unemployment insurance in a sufficient statistics formula that generalizes the result of [Chetty \(2008\)](#).

### 2.5.1 Welfare in terms of liquidity and moral hazard

As a first step, we study how infinitesimal changes in an unemployment insurance scheme characterized by  $(\bar{b}_1, \bar{b}_1, \tau)$  impact welfare. We start out as general as possible and take into account that marginal changes in benefit levels will also have an impact on  $\tau$  through fiscal externalities but do not (yet) specify the exact fiscal budget constraint. The following Lemma characterizes the conditions that need to hold for changes in benefit levels to be welfare-enhancing from an ex-ante perspective.

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<sup>8</sup>Although for any two consecutive periods  $k$  and  $k'$  the theory implies that  $\frac{MH_k}{LIQ_k} \geq \frac{MH_{k'}}{LIQ_{k'}}$ , exactly by how much this ratio decreases is an empirical question.

**Lemma 3** *Assume Assumptions 1 and 2 hold. Then infinitesimal changes in  $\bar{d}b_1$  and  $\bar{d}b_1$  raise ex-ante welfare of an unemployed worker, i.e.,  $dJ_0(\bar{b}_1, \bar{b}_2, \tau) \geq 0$  if and only if*

$$\sum_{k=1,2} \left[ -LIQ_k + MH_k \left( 1 - \frac{T-D}{D_k} \frac{d\tau}{d\bar{b}_k} \right) \right] d\bar{b}_k \geq 0. \quad (20)$$

Increasing the benefit level would unambiguously raise ex-ante welfare by an amount that is proportional to  $-(LIQ_k - MH_k)$  if it were costless to do so. However, because raising benefits has an associated fiscal cost, ex-ante welfare is reduced by a marginal cost term that is proportional to  $\frac{T-D}{D_k} MH_k \frac{d\tau}{d\bar{b}_k}$ . The term  $\frac{d\tau}{d\bar{b}_k}$  measures the marginal increase in taxes required to finance the marginal increase in benefits and the term  $MH_k$  expresses this concept in terms of marginal utility. Because taxes are paid only in the employed state and unemployed benefits collected only in the unemployed state, the marginal cost needs to be adjusted by  $\frac{T-D}{D_k}$ , the expected time spent in the employed versus in the unemployed state of the world.

The result in Lemma 3 shows that the marginal benefit of raising benefits in any period  $k$  is separable from the other period. However, the marginal cost is not. Different periods  $k$  are tied together through the fiscal budget constraint.

### 2.5.2 Fiscal externalities

We now consider the fiscal externalities that arise because of the existence of a budget constraint. To calculate the required increase in taxes per dollar of extra benefits  $\frac{d\tau}{d\bar{b}_k}$  it is necessary to specify the budget constraint faced by the planner.

We follow [Chetty \(2008\)](#) and consider the case in which the budget is balanced:

$$\tau(T - D) = \sum_k \bar{b}_k D_k = \bar{b}_1 D_1 + \bar{b}_2 D_2. \quad (21)$$

The result in Lemma 3 specifies the conditions under which the objective function increases for any arbitrary way of financing of unemployment benefits. In the balanced-budget case, the following Lemma is applicable.



**Lemma 4** *To satisfy a balanced budget, taxes respond to an increase in  $\bar{b}_k$  in the following way. For  $k = 1, 2$  and  $k' \neq k$ :*

$$\frac{d\tau}{d\bar{b}_k} = \frac{D_k}{T-D} \left[ 1 + \varepsilon_{D_k, \bar{b}_k} + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \varepsilon_{D_{k'}, \bar{b}_k} + \frac{D}{T-D} \left( 1 + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \right) \varepsilon_{D, \bar{b}_k} \right]. \quad (22)$$

The result in (22) directly follows from differentiating the budget constraint in (21). The economic intuition behind (22) is the following. If agents were not to modify their search effort in response to changes in the unemployment insurance scheme, then the expected increment in the cost of the scheme per dollar of increase in benefits would be equal to  $\frac{D_k}{T-D}$ , the expected time during which the benefits  $\bar{b}_k$  are received relative to the expected time during which the worker is employed and pays taxes. Therefore, absent moral hazard, recouping the cost of the policy explains only the presence of the first 1 inside the brackets in (22). The following two terms are due to fiscal externalities on the cost-side. The first of these terms is  $\varepsilon_{D_k, \bar{b}_k}$ , the elasticity of the duration  $D_k$  with respect to the benefit level  $\bar{b}_k$ . Because unemployed agents take into account  $\bar{b}_k$  when choosing search effort, the duration of their unemployment spell will be affected by changes in  $\bar{b}_k$ . The second term  $\frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \varepsilon_{D_{k'}, \bar{b}_k}$  is a cross-period elasticity as in [Kolsrud, Landais, Nilsson, and Spinnewijn \(2015\)](#); changes in  $\bar{b}_k$  can have an effect also on the duration of unemployment in a period in which the agent is not entitled to  $\bar{b}_k$  (in  $k' \neq k$ ).<sup>9</sup> The third term,  $\frac{D}{T-D} \left( 1 + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \right) \varepsilon_{D, \bar{b}_k}$ , is the fiscal externality on the revenue-side. The elasticity  $\varepsilon_{D, \bar{b}_k}$  measures the response of the total duration of unemployment (a time during which no taxes are collected) to changes in  $\bar{b}_k$ . This elasticity is multiplied by  $\frac{D}{T-D}$ , which takes into account the amount of time spent in unemployment relative to time employed and  $\frac{C(\mathbf{b})}{\bar{b}_k D_k} = 1 + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k}$ , which is the inverse of the cost spent on unemployment insurance in period  $k$  relative to the total cost of the insurance scheme  $C(\mathbf{b}) = \bar{b}_k D_k + \bar{b}_{k'} D_{k'}$ .

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<sup>9</sup>The benefit level  $\bar{b}_k$  can affect  $D_{k'}$  in two ways, according to whether period  $k$  lies in the future (and  $k' < k$ ) or in the past (and  $k' > k$ ). In the first of these cases, because agents are forward-looking, they modify their behavior in period  $k'$  in anticipation of changes in unemployment benefits in the future. In the second case, because they have made different choices in the past, and have modified the sequence of probabilities of finding a job at previous dates, they affect the probability of entering period  $k'$  in an unemployed state, therefore impacting the level of  $D_{k'}$ .

### 2.5.3 Optimal Unemployment Insurance

The problem solved by the planner is

$$\max_{\bar{b}_1, \bar{b}_2, \tau} J_0(\bar{b}_1, \bar{b}_2, \tau) \quad \text{subject to} \quad \tau(T - D) = \bar{b}_1 D_1 + \bar{b}_2 D_2 \quad (23)$$

By combining the results from Lemma 3 with those from Lemma 4 we obtain the condition for the optimality of an unemployment insurance scheme.

**Proposition 2** *Under Assumptions 1 and 2, if the unemployment insurance scheme is optimal, then, at an interior optimum, for any  $k$ :*

$$R_k \equiv -\frac{LIQ_k}{MH_k} = \varepsilon_{D_k, \bar{b}_k} + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \varepsilon_{D_{k'}, \bar{b}_k} + \frac{D}{T - D} \left( 1 + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \right) \varepsilon_{D, \bar{b}_k}. \quad (24)$$

This proposition is our main theoretical result on the normative side. It is a formula in the “sufficient statistics” tradition, which combines the liquidity and moral hazard effects with high-level elasticities in order to empirically assess whether unemployment benefits are at their optimal levels. The liquidity and moral hazard effects are identified off the beginning-of-spell hazard rates whereas the elasticities can be estimated from data that are directly observable. On the left hand side,  $R_k$  is the ratio of liquidity to moral hazard effects. If a higher fraction of the variation in hazard rate responses is explained by liquidity effects, then insurance becomes more valuable from the point of view of the social planner. Thus, the left hand side of the equation measures the benefits of increasing unemployment insurance. The right hand side measures the costs of raising the level of unemployment insurance. At the optimum, benefits and costs must coincide.

The formula we obtained generalizes the result obtained by Chetty (2008) for the case of a single benefit level. In fact, it can be shown that the formula simplifies to the formula by Chetty (2008) in the case with a single benefit level. A single benefit level can be represented in our environment by setting  $\bar{b}_1 = \bar{b}$  and  $\bar{b}_2 = 0$ . Doing so cancels two terms on the right hand side and simplifies the condition for optimality to

$$R = -\frac{LIQ}{MH} = \varepsilon_{D_B, \bar{b}} + \frac{D}{T - D} \varepsilon_{D, \bar{b}}. \quad (25)$$

Chetty further assumes that  $\varepsilon_{D_B, \bar{b}} = \varepsilon_{D, \bar{b}}$  for simplicity. Imposing this additional equal-

ity, our result simplifies to the result by [Chetty \(2008, eq. 14\)](#), according to which benefits are at their optimal level whenever

$$R = -\frac{LIQ}{MH} = \frac{T}{T-D}\varepsilon_{D,\bar{b}}. \quad (26)$$

Proposition 2 shows that if the environment of [Chetty \(2008\)](#) is generalized to more than one benefit level, then the optimality of each level can be separately determined using (24). On the left hand side, the formula features the ratios of liquidity to moral hazard effects over the two sub-periods over which benefits are constant, which can be identified from the data given the result in Proposition 1. On the right hand side, extra terms appear relative to the formula by [Chetty \(2008\)](#) because of the cross-effect of an increase in the level of benefits in one period on the costs of unemployment insurance in the other period.

### 3 Empirical Implementation: Strategy, Context and Data

#### 3.1 Empirical objects of interest

In order to give empirical content to our theoretical results, it is necessary to estimate the effect of unemployment benefit levels on a number of outcome variables. On the positive side (Proposition 1), to separate liquidity from moral hazard rates, the variables of interest are  $\frac{\partial s_0}{\partial b_1}$  and  $\frac{\partial s_0}{\partial b_2}$ : the effect on the hazard of exiting unemployment at the beginning of an unemployment spell of the different unemployment benefit levels. On the normative side (Proposition 2), it is necessary to obtain estimates of the effect of  $\bar{b}_1$  and  $\bar{b}_2$  on  $D_1$ ,  $D_2$ , and  $D$ : the expected unemployment duration while on benefits  $\bar{b}_1$  and  $\bar{b}_2$ , and the total expected unemployment duration.

#### 3.2 Empirical strategy

We estimate the effect of increasing benefit levels on the variables of interest by exploiting the piece-wise linear kinked relationship between pre-unemployment labor income and

the level of unemployment benefits. We exploit two kinks that arise due to a change in replacement rates during the unemployment spell. This strategy, termed the Regression Kink Design (RKD), is a close relative of a regression discontinuity design, and has been used in the context of unemployment benefits by [Landais \(2015\)](#) for the US, and by [Card, Lee, Pei, and Weber \(2015\)](#) for Austria. One of the advantages of the RKD is that the source of variation in unemployment benefits is time-invariant. In contrast, empirical strategies that use changes in legislation over time, face the potential pitfall that changes in legislation might be endogenous to labor market conditions.

### Regression Kink Design

In the RKD,  $Y$  is an outcome of interest,  $V$  is an observed variable (called the running variable) that affects  $Y$ , and  $B$  is the observed variable of interest. These variables are related according to a constant-effect additive model:

$$Y = \theta B + g(V) + \epsilon, \quad (27)$$

where  $B = g(V)$  is a deterministic and continuous function of  $V$  with a kink at  $V = 0$ . The logic of the RKD is that, given the kink in the relationship between  $V$  and  $B$ , if  $B$  affects  $Y$ , then there should also be a kink in the relationship between  $V$  and  $Y$  at that same point. In the RKD, the coefficient of interest  $\theta$  can be calculated as:

$$\theta = \frac{\lim_{v_0 \rightarrow 0^+} \left. \frac{dE[Y|V=v]}{dv} \right|_{v=v_0} - \lim_{v_0 \rightarrow 0^-} \left. \frac{dE[Y|V=v]}{dv} \right|_{v=v_0}}{\lim_{v_0 \rightarrow 0^+} g'(v_0) - \lim_{v_0 \rightarrow 0^-} g'(v_0)} \quad (28)$$

The numerator in (28) is the change in the slope in the conditional expectation function at the location of the kink, and the denominator is the change in the slope of the deterministic function  $g(V)$  at the kink. The value of the denominator does not need to be estimated. It is directly given by the administrative rule for the determination of the benefits. The numerator, on the other hand, is estimated by running a parametric model of the form:

$$E[Y|V=v] = \alpha + \eta'X + \gamma(v - \bar{v}) + \beta(v - \bar{v})W, \quad (29)$$

where  $Y$  and  $V$  are, as before, the outcome of interest and the running variable ( $V$  in our case is pre-unemployment labor income).  $X$  stands for additional covariates,  $\bar{v}$  is the level of the running variable at which the kink takes place, and  $W$  is a dummy variable that takes the value one for those observations above the kink and zero otherwise. This model is estimated for  $|v - \bar{v}| \leq h$ , where  $h$  is the bandwidth size. The numerator in (28) is captured by  $\beta$ .

In our case, given that there are two variables of interest, we write the equation to be estimated as:

$$Y = \theta_1 b_1 + \theta_2 b_2 + g(V) + \epsilon, \quad (30)$$

We estimate each  $\theta_k$  using a RKD in each of the kinks, and obtain for each outcome of interest the effect of the different unemployment benefit levels:  $b_1$  and  $b_2$ .

An assumption behind the RKD is that the direct effect of the running variable (pre-unemployment earnings) on the outcome of interest is smooth. Also, that unobserved heterogeneity does not change discontinuously at the kink in the running variable. Manipulation of the running variable would imply in our case that the worker is able to control labor earnings in the 180 working days prior to becoming unemployed. If this manipulation occurs, then it would imply a concentration of workers around the kinks. In Section 4 we discuss the validity of the RKD in our case in detail.

## Unemployment Benefits in Spain

In Spain, in order to be entitled to unemployment benefits a worker needs have worked for at least 360 working days in the six years before becoming unemployed. Once unemployed, the worker is entitled to receive unemployment benefits for a period that ranges from 120 to 720 days, depending on the length of the worker's prior employment spell. To obtain the maximum entitlement of 720 days it is necessary to have worked during 2,160 days. These days are not necessarily consecutive, and they are equivalent to six years in employment. The level of benefits is based on labor earnings in the 180 working days (registered at the Social Security Administration) prior to the onset of unemployment. For the period we analyze, the level of benefits is set at 70% of prior labor income during the first six months in unemployment, and at 60% during the remainder of the period in which the worker is entitled to unemployment benefits.

Benefits are capped below and above by values  $b_{min}$  and  $b_{max}$  that depend on an index

called IPREM, whose values are set by the government on a yearly basis. Minimum and maximum benefits are a function of IPREM and also on the worker having zero, one, or two or more dependents. A dependent is defined as someone who receives no income, lives with the person claiming unemployment, and is less than 26 years old or older than 26 but with a disability greater than 33%. The level of unemployment benefits depends on prior labor income  $V$  according to:

$$b_k = \begin{cases} b_{min} & \text{if } V \times r_k \leq b_{min} \\ V \times r_k & \text{if } b_{min} < V \times r_k \leq b_{max} , \\ b_{max} & \text{if } V \times r_k > b_{max} \end{cases} \quad k = 1, 2, \quad (31)$$

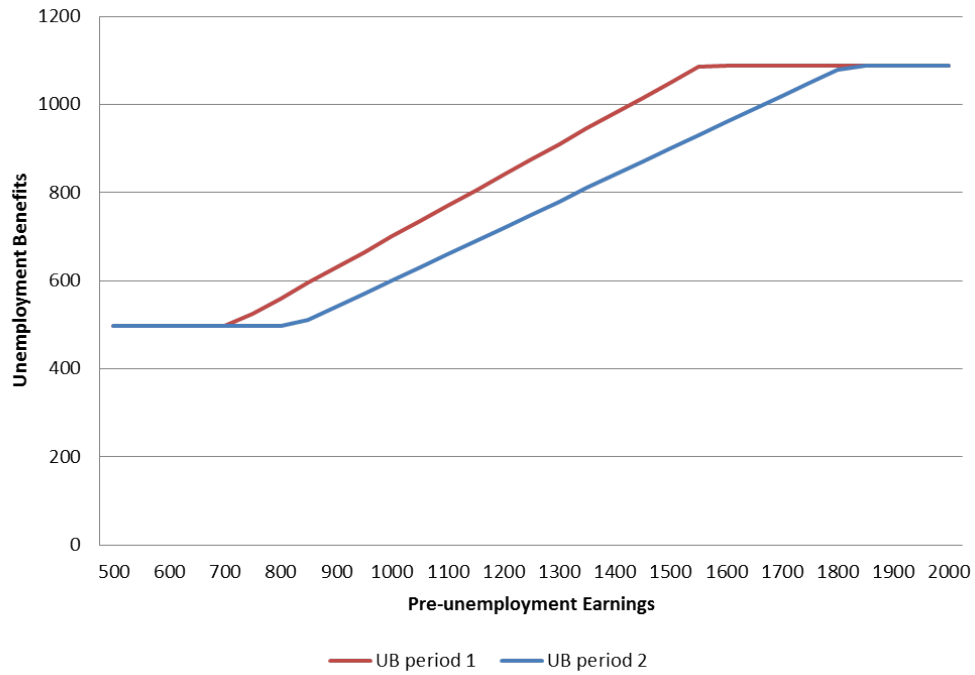
with  $b_{min}$  and  $b_{max}$  taking different values for different years and number of dependents, and  $r_1 = 70\%$  and  $r_2 = 60\%$ . As an example, in Figure 1 we plot unemployment benefits as a function of prior labor income for an individual without dependents using the value of the IPREM in 2011.

In our estimations we will use only the kink at  $b_{max}$ . The figure shows that the horizontal difference at  $b_{max}$  is larger than at  $b_{min}$ . Also, the number of workers of our sample at or close to the maximum kink is larger than at the minimum kink.

### 3.3 Data

Our data are from the Continuous Sample of Working Histories (Muestra Continua de Vidas Laborales, MCVL, in Spanish). This is a dataset based on administrative records provided by the Spanish Social Security Administration. Each wave contains a random sample of 4% of all the individuals who had contact with the Social Security system, either by working or by receiving a contributory benefit (such as unemployment insurance, permanent disability, old-age, etc.) during at least one day in the year the sample is selected.

The MCVL reconstructs the labor market histories of individuals in the sample back to 1967 (although earnings data are available only since 1980). Therefore, we have information on the entire labor history of the workers in the sample. Moreover, this dataset has a longitudinal structure from 2005 to 2014, meaning that an individual who is present in a wave and remains registered with Social Security (which is required to



**Figure 1:** *Unemployment Benefits as a function of pre-unemployment earnings in Spain*

*Note: We calculate unemployment benefits for an individual with no dependents using the value of the IPREM in 2011. The red line corresponds to the level of unemployment benefits in the first six months of unemployment. The blue line corresponds to unemployment benefits in the remainder of the unemployment spell.*

receive unemployment benefits) stays in the sample in subsequent waves. In addition, the sample is refreshed with new entrants, which guarantees the representativeness of the population in each wave. In our estimates we use the last ten waves (2005-2014), so that only those workers who were not registered with the Social Security Administration during at least one day in the period 2005-2014 are excluded from our sample.

There is information available on the entire employment, non-employment and pension history of the workers, including the exact duration of employment, non-employment and disability or retirement pension spells, and for each employment spell. Several variables that describe the characteristics of the job are present, such as the sector of activity, type of contract, number of hours, etc. There is also information on personal characteristics, such as age, gender, nationality, and level of education, although this information is only kept current starting in 2005. Periods of non-employment are identified using information on the dates in which the firm does not pay Social Security contributions for the worker. Those non-employment spells in which the worker receives unemployment benefits are clearly identified as unemployment spells. Given that the dataset contains all the social security payments made by the firms, we can compute the exact entitlement to unemployment benefits for each unemployment spell and the level of unemployment benefits also for workers who switched jobs.

In our regressions we use all available waves of the MCVL but restrict the sample to unemployment spells starting between 2005 and 2011. We start the period in 2005 because for prior years there are no contemporaneous data on dependents, which is a necessary input to calculate the exact location of the kink in the benefits schedule. We stop the period in 2011 because the fractions of the pre-unemployment earnings used to compute unemployment benefits are constant until 2011 at 70% in the first six months and 60% during the subsequent 18 months. With this restriction we ensure that there are no changes in the institutional framework. We consider only spells after full-time employment and in the general regime. We further restrict the sample to individuals who are aged between 30 and 50 and who are entitled to the maximum amount of unemployment benefits.<sup>10</sup> By using only individuals with the maximum level of benefits (the largest group in the sample) in our baseline analysis we ensure that we apply the results of Proposition 1 to a homogeneous sample. We later relax this last requirement in a robustness check.

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<sup>10</sup>We exclude workers older than 50 because in Spain they are eligible for subsidies that provide incentives for those workers to stay out of the labor force until they can legally retire.



**Table 1:** *Descriptive Statistics: spells in main regression sample*

	Mean	SD
<i>Duration</i>		
Entitlement	720.00	(0.00)
Total duration	358.74	(275.86)
Duration Period 1	138.74	(60.78)
Duration Period 2	220.31	(232.92)
Exhaustion	0.33	(0.47)
Exit during period 1	0.38	(0.49)
<i>Earnings</i>		
UB period 1	1,131.08	(169.46)
UB period 2	1,063.23	(204.94)
Pre-unemployment Earnings	2,256.43	(975.91)
Fraction with max UB in period 1	0.68	(0.47)
Fraction with max UB in period 2	0.52	(0.50)
<i>Covariates</i>		
Age	40.73	(5.87)
Dependents	0.70	(0.90)
Male	0.72	(0.45)
Obs.	8,635	

*Note: Entitlement is the number of days that a worker is entitled to receive unemployment benefits. Duration represents the number of days spent in unemployment. Period 1 corresponds to the first six months of the unemployment spell, and Period 2 to the subsequent 18 months. Exhaustion is a dummy taking the value one if the worker exhausts her benefits. Exit during period 1 is a dummy that takes the value one if the worker leaves unemployment during the first six months. Pre-unemployment earnings represents average monthly earnings in the previous 180 working days. UB denotes unemployment benefits. All monetary values are expressed in real terms.*

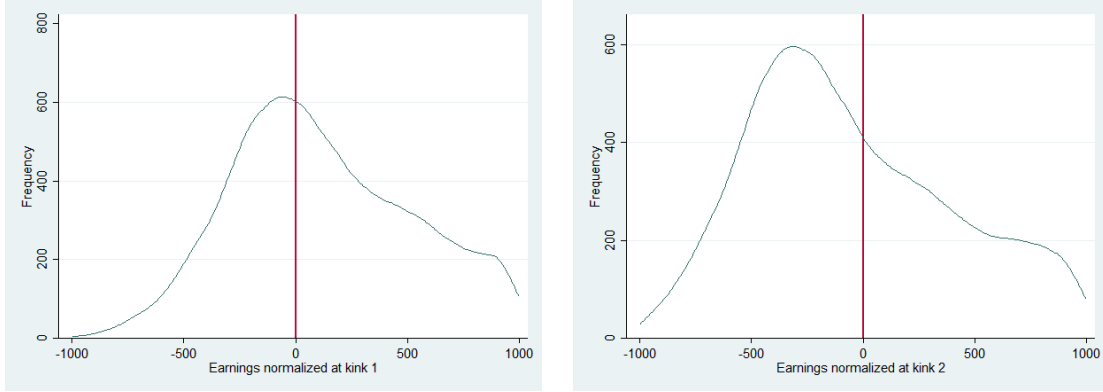
Table 1 contains descriptive statistics of the unemployment spells used in our regressions. There are 8,635 unemployment spells that satisfy the above criteria. We report the mean and standard deviation on variables related to unemployment duration, earnings, and additional variables that we use as covariates in our estimations. On average, unemployment spells in our sample last about 359 days, 139 days in the first period of six months, and an extra 220 days in the second period of 18 months. In addition to observing long durations of unemployment, we also observe an important fraction of unemployed individuals exhausting their benefits (33% of the spells last the maximum possible duration). On the other hand, around 38% of the unemployed exit unemployment in the first six months. Average pre-unemployment earnings are EUR 2,256 (in constant 2011 euros) and average unemployment benefits hover around 1,130 in the first period and 1,060 in the second period. There is a substantial number of unemployed with labor earnings that place them at the maximum benefit level: 68% in the first period and 52% in the second period, ensuring that there are sufficient observations on both sides of the kink. The average age is 40.7 years, 72% of the sample consists of males, and the average number of dependents is 0.7.

## 4 Estimation results

### 4.1 Graphical Evidence

We present graphical evidence in support of the application of a RKD, following the testable propositions proposed by [Card, Lee, Pei, and Weber \(2015\)](#). The key identifying assumption is the smoothness of the density of the running variable: pre-unemployment earnings. This assumption will not hold if we observe a discontinuity in the density of pre-unemployment earnings around the kinks. In [Figure 2](#) we plot the distribution of pre-unemployment earnings normalized at each kink. We do not observe any type of discontinuity around the kinks. In addition, in [Figure 3](#) we perform a discontinuity test based on [McCrary \(2008\)](#). We plot the probability density function of the running variable to show the smoothness of the distribution of pre-unemployment earnings at both kink points. This smoothness is evidence against the possibility of manipulation of earnings at the kink point. Because we normalize earnings dividing them by each kink, both graphs present the kink at one, regardless the year or the kink. Both figures also

include results of a McCrary test that reinforce the conclusion of no manipulation of the running variable at the kinks.<sup>11</sup>



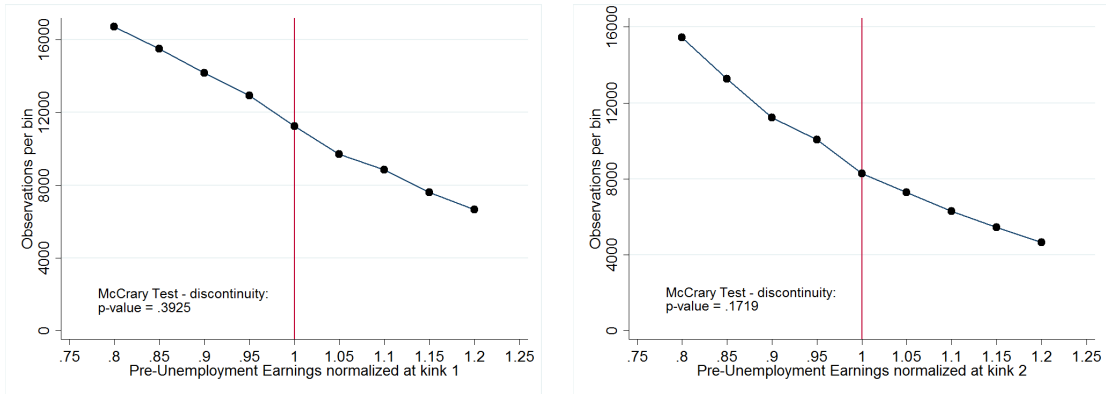
**Figure 2:** *Frequency distribution of pre-unemployment earnings around the kinks*

*Notes: The figures show frequency distribution of pre-unemployment earnings normalized at each of the kinks. These figures graphically show the smoothness of the distribution of the running variable.*

The second testable assumption is that the conditional distribution function of predetermined variables is smooth at the kinks. In Figure 4 we plot covariates and their relationship with the running variable. We plot the mean of age, number of dependents, and gender in each bin of the running variable. The graphs provide evidence on the smoothness in the relationship between these covariates and pre-unemployment earnings at both kinks, with no jumps at any of the kinks.

Finally, we look at three outcomes of interest: the probability of exiting unemployment in the first 6 months, total duration of the unemployment spell, and duration in non-employment. In Figures 5 and 6 we verify the existence of discontinuities in the graphs at both kinks. The first of these graphs shows the mean values of a given outcome in a bin of pre-unemployment earnings normalized at each kink. The second graph shows the fit of a piecewise-linear regression between the outcome of interest and pre-unemployment earnings normalized at each kink. This analysis provides visual evidence on the relationship between the running variable and the outcomes of interest. Although the figures do not take into account the effect of other covariates, and therefore do not

<sup>11</sup>The implementation of this test is based on the tests of manipulation of the running variable for RD designs presented in McCrary (2008), and implemented for the case of RKD in Landais (2015).

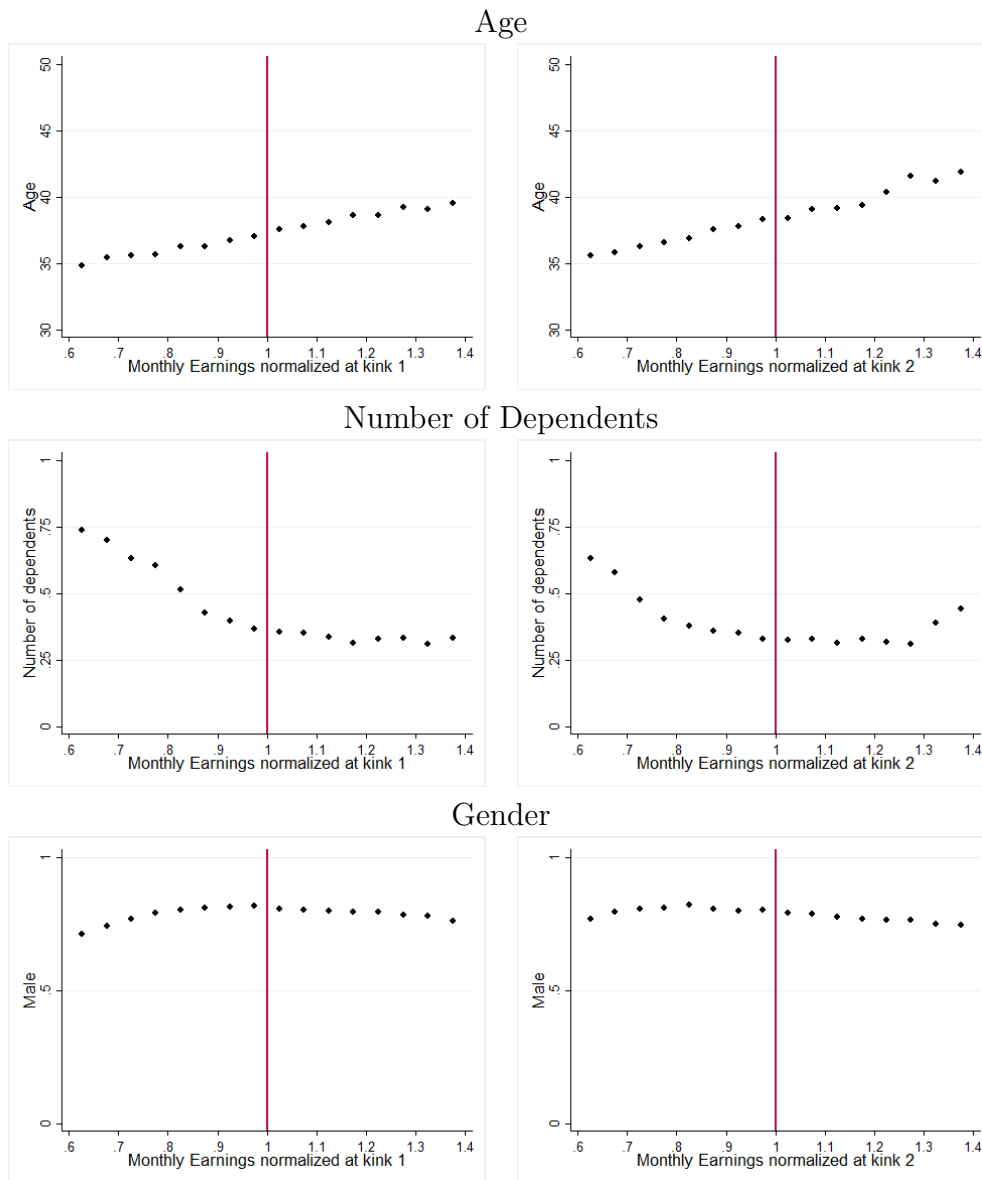


**Figure 3:** *Frequency distribution of pre-unemployment earnings around the kinks*

*Notes: The figures show frequency distribution of pre-unemployment earnings normalized at each of the kinks. These figures graphically show the smoothness of the distribution of the running variable. We include the p-value for a McCrary test which null hypothesis of continuity cannot be rejected.*

show directly the effect of benefit levels on the variables of interest, it is especially interesting to note the existence of jumps and discontinuities in the graphs for the outcome variables in contrast to the absence of them in the graphs for the covariates.<sup>12</sup>

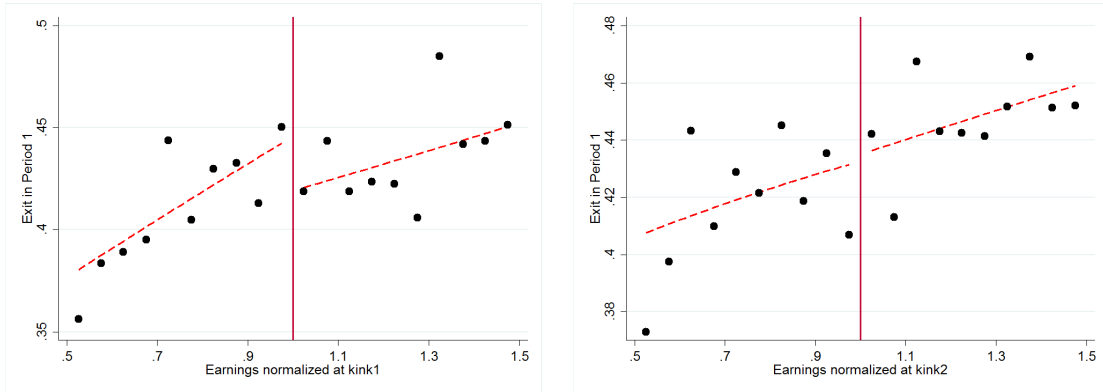
<sup>12</sup>The jumps are particularly prominent in Figure 6, where the estimation of a piecewise-linear relationship mimics the piecewise-linear specification used in the baseline regressions.



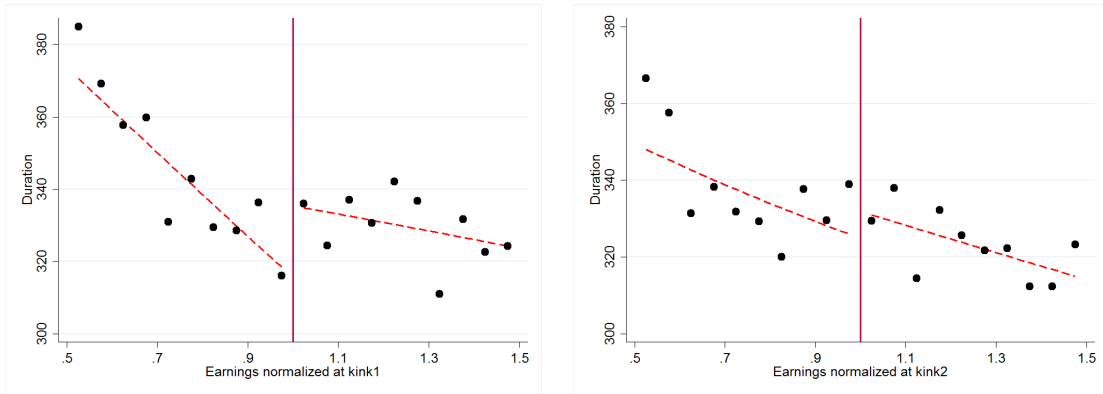
**Figure 4:** *Covariates and earnings*

*Note: Each figure shows the mean values of the given covariate in a bin of pre-unemployment earnings normalized at each kink. These graphs give a visual validation of the assumption of smoothness around the kinks.*

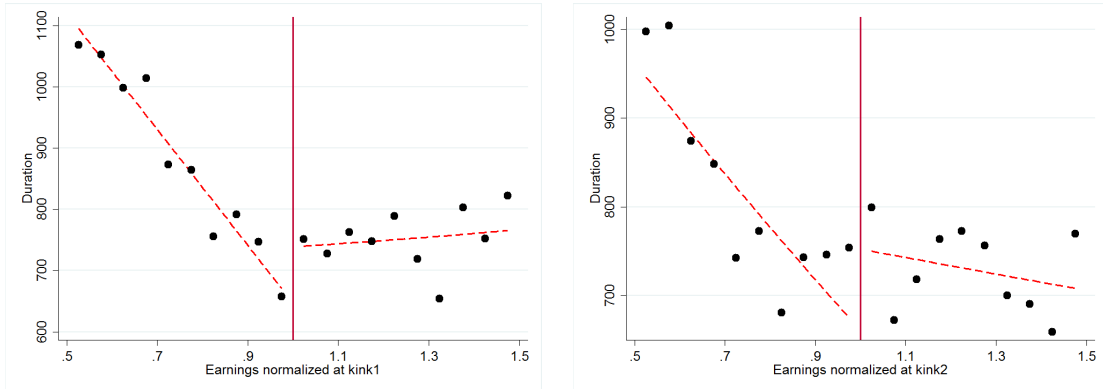
### Probability of leaving unemployment



### Unemployment duration



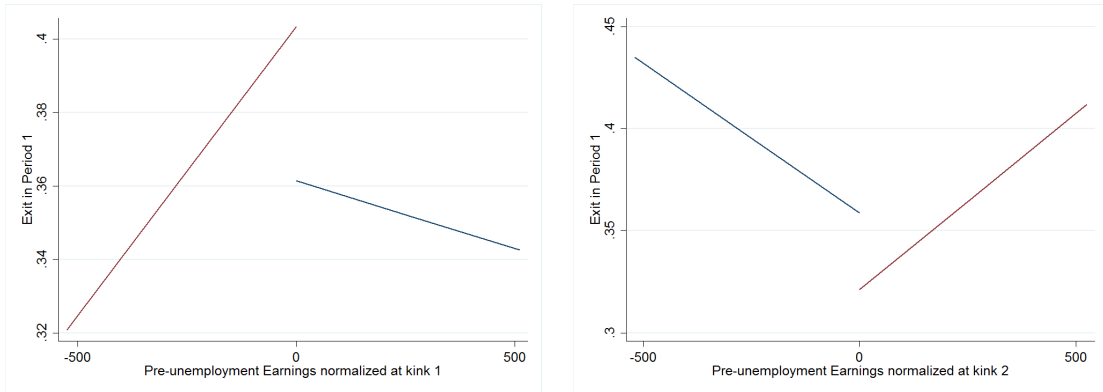
### Non-employment duration



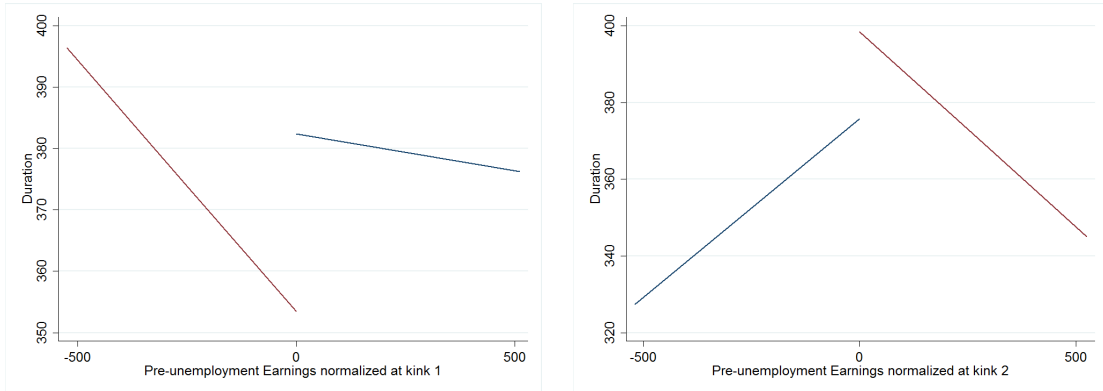
**Figure 5:** Probability of exiting unemployment in the first six months, unemployment duration, and non-employment duration: mean values of a given outcome in a bin of pre-unemployment earnings normalized at each kink

Note: Each figure shows the piecewise-linear relationship between each outcome and mean values of the bins of pre-unemployment earnings normalized at each kink.

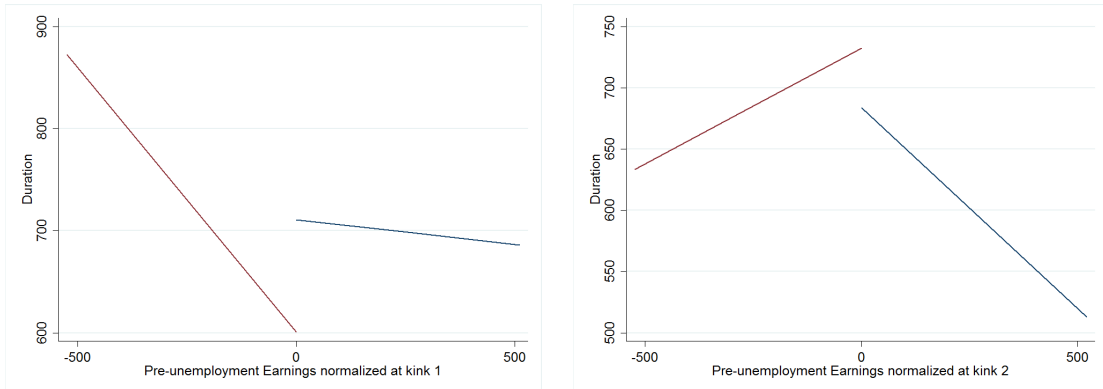
### Probability of leaving unemployment



### Unemployment duration



### Non-employment duration



**Figure 6:** Probability of exiting unemployment in the first six months, unemployment duration, and non-employment duration: relationship with pre-unemployment earnings

*Note: Each figure shows the piecewise-linear relationship between each outcome and pre-unemployment earnings normalized at each kink.*

## 4.2 Results

We estimate models controlling for year dummies, age at the time of becoming unemployed, and this age squared, and dummies for the the number of dependents, gender, having a permanent contract in the previous job, the number of prior unemployment spells, qualifications of the job, and regions. We choose  $h = 450$  for the bandwidth size and later check the robustness to other bandwidth choices. Results for the variables of interest are presented in Table 2. We transform the coefficients obtained in the regressions into the marginal impact of increasing benefits in each one of the two periods on each outcome according to the formula for  $\theta$  in (28):  $\theta_1$  represents the impact of increasing benefits in the first six-months period, and  $\theta_2$  the impact of increasing benefits in the second period.<sup>13</sup>

The probability of exiting unemployment in the first six-months (our measure for  $s_0$ ) decreases with higher unemployment benefits. Coefficients in first column are multiplied by 100, so that an EUR 100 increase in  $\bar{b}_1$ , the level of unemployment benefits in the first period, implies a decrease of around 4.5 percentage points in  $s_0$ . In turn, an EUR 100 increase in  $\bar{b}_2$  implies a decrease of around 5.5 percentage points in  $s_0$ . The second column in Table 2 shows that unemployment duration in the first six months also increases with unemployment benefits:  $D_1$  increases on average by 4.5 days per EUR 100 increase in  $\bar{b}_1$  and by 4.9 days per EUR 100 increase in  $\bar{b}_2$ . Unemployment duration in the second period,  $D_2$ , increases on average by 18 and 29 additional days per EUR 100 increase in  $\bar{b}_1$  and  $\bar{b}_2$ . Finally, total non-employment duration  $D$  increases by 68 and 101 additional days per EUR 100 increase in unemployment benefits in periods 1 and 2. Point estimates are all of the expected sign and significantly different from zero.

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<sup>13</sup>Note that the dependent variable in the first column in Table 2 is in both cases a dummy variable that takes the value one if the worker exits unemployment in the first six months (Period 1) and zero otherwise. In both estimations, for  $\theta_1$  and  $\theta_2$ , we use all workers in the sample around each kink regardless the actual duration of the unemployment spell, therefore these estimates are not affected by selection bias.



**Table 2:** *RKD estimations on several outcomes: Period 2005 - 2012, workers between 30 and 50 years old*

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	Exit in Period 1	Duration Period 1	Duration Period 2	Total Duration	MH	Optimal
$\theta_1$	-0.045*** (0.016)	0.045** (0.021)	0.182** (0.080)	0.677** (0.286)	70%	Too high
Observations	3,751	3,751	3,751	3,669		
	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	Exit in Period 1	Duration Period 1	Duration Period 2	Total Duration	MH	Optimal
$\theta_2$	-0.055*** (0.021)	0.049* (0.025)	0.286*** (0.097)	1.010*** (0.320)	25%	Too high
Observations	3,422	3,422	3,422	3,346		

*Note: All estimates from models controlling for year dummies, age (at the time of becoming unemployed) and age squared, dummies for having one or more than one dependents, a dummy for having a permanent contract in the previous job, dummies for the qualifications of the job, for the number of the unemployment spell, and dummies for provinces. Duration in each period is days in unemployment in each period. Total duration is days in non-employment. Coefficients are transformed in order to obtain the values of interest: the impact of increasing benefits in each period on each outcome.*

### 4.2.1 Liquidity and moral hazard

The estimations in Table 2 yield an estimate of  $\frac{\partial s_0}{\partial \bar{b}_1} = -0.045$  and  $\frac{\partial s_0}{\partial \bar{b}_2} = -0.055$ . Using the formulas in Proposition 1 we can separate these effects into a liquidity effect and a moral hazard effect. We define a period as lasting six months. With this convention,  $B_1 = 1$  and  $B_2 = 3$ . To make results representative of the whole population, we use the durations  $D_1$  and  $D_2$  for the entire sample rather than those for our selected subsample and set  $D_1 = \frac{160.76}{180} = 0.89$  and  $D_2 = \frac{71.44}{180} = 0.40$ .<sup>14</sup> Plugging these values into the formulas of Proposition 1, we find that

$$\frac{\partial s_0}{\partial \bar{b}_1} = -0.045 = \underbrace{-0.0136}_{LIQ_1} - \underbrace{0.0315}_{MH_1} \quad (32)$$

and

$$\frac{\partial s_0}{\partial \bar{b}_2} = -0.055 = \underbrace{-0.0409}_{LIQ_2} - \underbrace{0.0140}_{MH_2} \quad (33)$$

Our estimations imply that over the first 6-month period, the liquidity effect accounts for 30% of the total effect whereas the moral hazard effect accounts for 70%. Over the period during which unemployment benefits are at  $\bar{b}_2$  the situation is reversed: liquidity and moral hazard effects account, respectively, for 75% and 25% of the total response of the 6-month hazard rate. In consequence, the ratios of liquidity to moral hazard effects, which play a key role in the normative results of Proposition 2, are estimated at  $R_1 = \frac{30\%}{70\%} = 0.43$  and  $R_2 = \frac{75\%}{25\%} = 2.92$ .

In comparison, Chetty (2008) finds that the liquidity effect accounts for 60% of the total effect of benefits on job search. The ratio of liquidity to moral hazard effects estimated by Chetty is therefore  $R = \frac{60\%}{40\%} = 1.5$  and lies between our estimates for the two periods. Landais (2015) reports a lower ratio of liquidity to moral hazard effects of  $R = 0.9$ , also in the range of our results, implying that approximately 47% of the total effect corresponds to the liquidity effect.

Our estimations in Table 2 also yield results on the fiscal cost of raising unemployment benefits. The fiscal externalities of raising  $\bar{b}_1$  are captured by  $\varepsilon_{D_1, \bar{b}_1} = 0.39$ ,  $\varepsilon_{D_2, \bar{b}_1} = 0.97$ , and  $\varepsilon_{D, \bar{b}_1} = 1.18$ . In turn, the fiscal externalities of raising  $\bar{b}_2$  are captured by the

<sup>14</sup>The implicit assumption we are making is that the relative importance of the responses of hazard rates in the subsample are similar to those that would be obtained for the whole population. We revisit this point later.

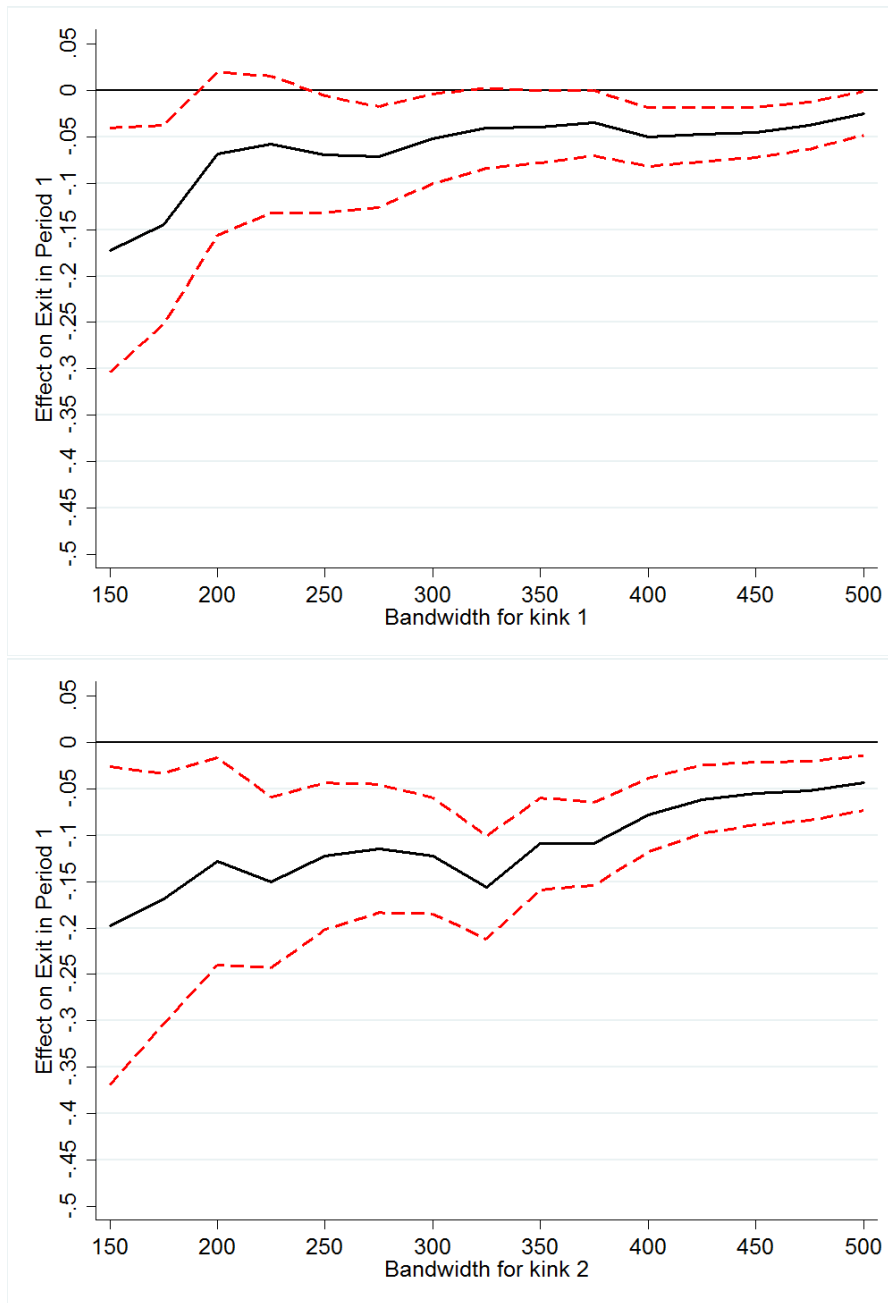
estimated elasticities  $\varepsilon_{D_1, \bar{b}_2} = 0.43$ ,  $\varepsilon_{D_2, \bar{b}_2} = 1.53$ , and  $\varepsilon_{D, \bar{b}_2} = 1.76$ . The elasticities for total duration in Spain are on the high side compared to the value of  $\varepsilon_{D, \bar{b}} = 0.5$  assumed for the US based on the survey by [Krueger and Meyer \(2002\)](#). For Spain, [Rebollo-Sanz and Rodríguez-Planas \(2015\)](#) find an elasticity of unemployment duration to the replacement rate ( $\varepsilon_{D, r}$ ) of 0.86, although for a different period. For Sweden, [Kolsrud, Landais, Nilsson, and Spinnewijn \(2015\)](#) estimate  $\varepsilon_{D, \bar{b}} = 1.53$ ,  $\varepsilon_{D_1, \bar{b}} = 1.32$ , and  $\varepsilon_{D_2, \bar{b}} = 1.62$  for a joint increase in  $\bar{b}_1$  and  $\bar{b}_2$  and, using 2001 data,  $\varepsilon_{D, \bar{b}_2} = 0.68$ ,  $\varepsilon_{D_1, \bar{b}_2} = 0.60$ , and  $\varepsilon_{D_2, \bar{b}_2} = 0.59$ . Although there are differences in context and time, the comparison with these other studies suggests that our estimates for the elasticities of durations are in a plausible range.

#### 4.2.2 Robustness checks

Our estimates are robust to the use of different bandwidths. In [Figure 7](#) we plot the point estimates for the probability of exiting unemployment in the first period for different bandwidths along with 95% confidence intervals. Results do not vary much with the bandwidth choice. In particular, the relative importance of liquidity and moral hazard effects remains remarkably constant.

In our baseline estimations we use only spells of workers who were entitled to 720 days of unemployment benefits. This restriction is necessary to have a homogeneous population. The problem of mixing workers entitled to different lengths of coverage is that for those with shorter coverage, a change in the level of unemployment benefits impacts workers over a shorter period, and we should therefore expect to find a weaker response. However, because our objective is to learn about the population as a whole, it is useful to increase the sample and include workers with shorter entitlements, keeping the previous caveat in mind. In [Table 3](#) we show the results of progressively lifting the restriction by adding spells with shorter entitlements.

As we move through the columns to the right in [Table 3](#) we find that the effect of the level of benefits on  $s_0$  decreases and that the precision of the estimation does not increase despite the addition of more observations. Despite these changes in the point estimates, the liquidity and moral hazard effects calculated from these estimates are remarkably stable. The main conclusion, that most of the total effect is due to moral hazard in the first period and to the liquidity effect in the second period, is unaffected by the addition of spells with shorter potential duration.



**Figure 7:** Estimates on the probability of exiting unemployment in the first period for different bandwidths, with 95% confidence intervals.

**Table 3:** *RKD estimations for different entitlements: 2005-2012, workers between 30-50 years old*

	(1)	(2)	(3)	(4)
VARIABLES	Entitlement 720	Entitlement at least 660	Entitlement at least 600	Entitlement at least 540
$\theta_1$	-0.045*** (0.016)	-0.031** (0.013)	-0.027** (0.012)	-0.022* (0.012)
Observations	3,751	6,170	6,858	7,643
MH	70%	80%	78%	82%
Optimality	High	High	High	High
	(1)	(2)	(3)	(4)
VARIABLES	Entitlement 720	Entitlement at least 660	Entitlement at least 600	Entitlement at least 540
$\theta_2$	-0.055*** (0.018)	-0.030* (0.014)	-0.027* (0.013)	-0.020 (0.013)
Observations	3,422	5,507	6,033	6,644
MH	25%	37%	35%	40%
Optimality	High	High	High	High

*Note: All estimates from models controlling for year dummies, age (at the time of becoming unemployed) and age squared, dummies for having one or more than one dependents, a dummy for having a permanent contract in the previous job, dummies for the qualifications of the job, for the number of the unemployment spell, and dummies for provinces. Coefficients are transformed in order to obtain the values of interest: the impact of increasing benefits in each period on each outcome.*

### 4.3 Optimal Unemployment Insurance: Calibration for Spain

Armed with our estimates we now attempt to shed light on whether  $\bar{b}_1$  and  $\bar{b}_2$  are set at their optimal levels. Our results for hazard rates yielded estimates of  $R_1$  and  $R_2$ . These numbers have to be compared to the right hand side of the expression in Proposition 2. Assuming that in the long term the ratio of time spent in unemployment to time spent working  $\frac{D}{T-D} = 0.10$ , and given our estimated elasticities, the right hand side for  $k = 1$  is calculated at 0.96, of which 0.79 is due to the rise in expected costs and, the remainder, 0.17 is the drop in expected revenue arising from an increase in  $\bar{b}_1$ . For  $k = 2$ , the expected marginal cost of raising unemployment benefits is estimated at 3.19, of which 2.58 is due to the expected rise in costs and 0.61 is due to the fall in revenues:

$$R_1 = 0.43 < 0.96 = \underbrace{\varepsilon_{D_1, \bar{b}_1} + \frac{\bar{b}_2 D_2}{\bar{b}_1 D_1} \varepsilon_{D_2, \bar{b}_1}}_{0.79} + \underbrace{\frac{D}{T-D} \left(1 + \frac{\bar{b}_2 D_2}{\bar{b}_1 D_1}\right) \varepsilon_{D, \bar{b}_1}}_{0.17} \quad (34)$$

and

$$R_2 = 2.92 < 3.19 = \underbrace{\varepsilon_{D_2, \bar{b}_2} + \frac{\bar{b}_1 D_1}{\bar{b}_2 D_2} \varepsilon_{D_1, \bar{b}_2}}_{2.58} + \underbrace{\frac{D}{T-D} \left(1 + \frac{\bar{b}_1 D_1}{\bar{b}_2 D_2}\right) \varepsilon_{D, \bar{b}_2}}_{0.61}. \quad (35)$$

Given our point estimates for Spain, marginal costs exceed the marginal benefits of raising  $\bar{b}_1$  and that therefore optimal unemployment insurance would lower  $\bar{b}_1$ . In the case of  $\bar{b}_2$ , the marginal benefit of raising unemployment insurance is higher than for  $\bar{b}_1$  because a larger part of the total effect on the hazard rate is due to the liquidity effect but this is counteracted by higher costs, in part due to the large estimate of  $\varepsilon_{D_2, \bar{b}_2}$ . According to point estimates,  $\bar{b}_2$  is therefore also set too high, as marginal costs exceed marginal benefits.

However, marginal benefits and marginal costs are more similar in the case of  $\bar{b}_2$ . The ratios used in our analysis are constructed using the point estimates in Table 2. In order to incorporate the uncertainty from those estimations, we bootstrap standard errors using 5,000 replications to obtain the empirical distribution for  $R_1$  and  $R_2$ . Using these empirical distributions, we test the hypothesis that  $R_k$  is equal to the right hand side of the expression in Proposition 2, against the alternative that  $R_k$  is lower (implying that optimal  $\bar{b}_k$  is lower). For the first period, we strongly reject the null hypothesis ( $p = 0.0006$ ), in favor of the alternative hypothesis that benefit levels are too high. For

the second period, we cannot reject the null hypothesis ( $p = 0.2316$ ). We therefore do not reject the null hypothesis that unemployment benefits  $\bar{b}_2$  are set at the optimal level.

Through the lens of our model, our calibration for Spain implies that over the period 2005–2011 benefit levels were too high in the first 6 months of the unemployment spell and approximately optimal thereafter. Because benefits in the period we consider decrease from 70% of prior labor income in the first 6 months to 60% in the subsequent period, this implies a benefit schedule that does not decrease so markedly. Moreover, according to the results of our model, the change in benefits in the 2012 labor reform, which decreased benefit levels over the second period from 60% to 50%, and made the benefit schedule steeper, was not a welfare-improving change. It reduced liquidity of the unemployed at the moment in the unemployment spell when it was most valuable and the size of moral hazard costs and fiscal externalities was not high enough to counteract this positive effect of unemployment insurance.

## 5 Conclusion

In this paper we study unemployment insurance schemes with time-varying benefits. We make two theoretical contributions. Our first theoretical contribution is to show that an insurance scheme in which unemployment benefits vary during the unemployment spell, as is the case in Spain, where higher benefits are paid during the first six months and drop afterwards, provides the necessary variation in the data to separately identify the moral hazard and liquidity effects of [Chetty \(2008\)](#). Our second theoretical contribution is to derive a “sufficient statistics” formula which, using the separation into liquidity and moral hazard effects allows us to verify whether the benefits at each of their time-varying levels are set at their optimal level.

We use administrative data from Spain (the MCVL), and a Regression King Design to obtain the estimates needed to disentangle liquidity and moral hazard effects. We then feed these estimates into the formula for the optimal benefit level. Our preliminary findings indicate that during the first six months moral hazard effects dominate and that the benefits of unemployment insurance are low relative to the costs. On the other hand, after the initial six months, when benefits in the Spanish system are lower, liquidity effects explain about three quarters of the change in hazard rates, raising the value of providing insurance. Given our estimates for the elasticities of unemployment duration

with respect to the benefit level paid at this later stage, we find that the benefit level in this second period is close to optimal, implying that liquidity effects warrant a schedule of benefits that does not decrease as markedly as it does in the Spanish system.

Our model is admittedly stylized and does not include general equilibrium effects. This calls for caution when using it for public policy. However, we hope that the ease of applying the formula using just data on unemployment spells and the novel identification of liquidity and moral hazard effects will earn it a place among the numerous tools in the arsenal of policymakers.



## References

- ARELLANO, M., S. BENTOLILA, AND O. BOVER (2004): “Paro y Prestaciones: Nuevos Resultados para España,” in *Estudios en Homenaje a Luis Ángel Rojo*, ed. by J. Pérez, C. Sebastián, and P. Tedde, vol. 1 (Políticas, mercados e instituciones económicas). Editorial Complutense, Madrid.
- BAILY, M. N. (1978): “Some aspects of optimal unemployment insurance,” *Journal of Public Economics*, 10(3), 379–402.
- BARCELÓ, C., AND E. VILLANUEVA (2016): “The response of household wealth to the risk of job loss: Evidence from differences in severance payments,” *Labour Economics*, 39(C), 35–54.
- BOVER, O., M. ARELLANO, AND S. BENTOLILA (2002): “Unemployment Duration, Benefit Duration and the Business Cycle,” *Economic Journal*, 112(479), 223–265.
- CAMPOS, R. G., AND I. REGGIO (2015): “Consumption in the shadow of unemployment,” *European Economic Review*, 78, 39 – 54.
- CARD, D., D. S. LEE, Z. PEI, AND A. WEBER (2015): “Inference on Causal Effects in a Generalized Regression Kink Design,” *Econometrica*, 83(6), 2453–2483.
- CHETTY, R. (2006): “A general formula for the optimal level of social insurance,” *Journal of Public Economics*, 90(10-11), 1879–1901.
- (2008): “Moral Hazard versus Liquidity and Optimal Unemployment Insurance,” *Journal of Political Economy*, 116(2), 173–234.
- HOPENHAYN, H. A., AND J. P. NICOLINI (1997): “Optimal Unemployment Insurance,” *Journal of Political Economy*, 105(2), 412–38.
- KOLSRUD, J., C. LANDAIS, P. NILSSON, AND J. SPINNEWIJN (2015): “The Optimal Timing of Unemployment Benefits: Theory and Evidence from Sweden,” IZA Discussion Papers 9185, Institute for the Study of Labor (IZA).
- KRUEGER, A. B., AND B. D. MEYER (2002): “Labor Supply Effects of Social Insurance,” Working Paper 9014, National Bureau of Economic Research.

- LANDAIS, C. (2015): “Assessing the Welfare Effects of Unemployment Benefits Using the Regression Kink Design,” *American Economic Journal: Economic Policy*, 7(4), 243–78.
- LENTZ, R., AND T. TRANAES (2005): “Job Search and Savings: Wealth Effects and Duration Dependence,” *Journal of Labor Economics*, 23(3), 467–490.
- MCCRARY, J. (2008): “Manipulation of the running variable in the regression discontinuity design: A density test,” *Journal of Econometrics*, 142(2), 698–714.
- NIELSEN, H. S., T. SØRENSEN, AND C. TABER (2010): “Estimating the Effect of Student Aid on College Enrollment: Evidence from a Government Grant Policy Reform,” *American Economic Journal: Economic Policy*, 2(2), 185–215.
- REBOLLO-SANZ, Y. F., AND N. RODRÍGUEZ-PLANAS (2015): “When the Going Gets Tough... Financial Incentives, Duration of Unemployment and Job-Match Quality,” unpublished.
- SHIMER, R., AND I. WERNING (2008): “Liquidity and Insurance for the Unemployed,” *American Economic Review*, 98(5), 1922–42.

## Appendix A: Proofs

**Lemma 1.** Implicit differentiation the first order condition for search  $s_t$  in (10) implies that for any  $x \in \{a_{t+j}, b_{t+j}, w_{t+j}\}$ :

$$\frac{\partial s_t}{\partial x} = \frac{1}{\psi''(s_t)} \left[ \frac{\partial V_t(A_t)}{\partial x} - \frac{\partial U_t(A_t)}{\partial x} \right] \quad (36)$$

Recursively substituting the maximized expressions for  $V$ ,  $U$  and  $J$  we obtain the effect in any period  $t$  of raising benefits and wages in period  $t + j$ ,  $j \geq 0$ :

$$\begin{aligned} \frac{\partial V_t(A_t)}{\partial b_{t+j}} &= 0, \quad j \geq 0 \\ \frac{\partial U_t(A_t)}{\partial b_{t+j}} &= \begin{cases} u'(c_t^u) & \text{if } j = 0 \\ u'(c_{t+j}^u) \beta^j \prod_{i=1}^j (1 - s_{t+i}) & \text{if } j \geq 1 \end{cases} \end{aligned} \quad (37)$$

$$\begin{aligned}\frac{\partial V_t(A_t)}{\partial w_{t+j}} &= v'(c_{t+j}^e)\beta^j, \quad j \geq 0 \\ \frac{\partial U_t(A_t)}{\partial w_{t+j}} &= \begin{cases} 0 & \text{if } j = 0 \\ v'(c_{t+j}^e)\beta^j \left(1 - \prod_{i=1}^j (1 - s_{t+i})\right) & \text{if } j \geq 1 \end{cases}\end{aligned}\quad (38)$$

For  $a_t$  we can use that  $\frac{\partial V_t(A_t)}{\partial a_{t+j}} = \frac{\partial V_t(A_t)}{\partial b_{t+j}} + \frac{\partial V_t(A_t)}{\partial w_{t+j}}$  and  $\frac{\partial U_t(A_t)}{\partial a_{t+j}} = \frac{\partial U_t(A_t)}{\partial b_{t+j}} + \frac{\partial U_t(A_t)}{\partial w_{t+j}}$  and therefore:

$$\begin{aligned}\frac{\partial V_t(A_t)}{\partial a_{t+j}} &= v'(c_{t+j}^e)\beta^j, \quad j \geq 0 \\ \frac{\partial U_t(A_t)}{\partial a_{t+j}} &= \begin{cases} u'(c_t^u) & \text{if } j = 0 \\ v'(c_{t+j}^e)\beta^j \left(1 - \prod_{i=1}^j (1 - s_{t+i})\right) + u'(c_{t+j}^u)\beta^j \prod_{i=1}^j (1 - s_{t+i}) & \text{if } j \geq 1 \end{cases}\end{aligned}\quad (39)$$

Using these results in (36):

$$\frac{\partial s_t}{\partial b_{t+j}} = \begin{cases} -\frac{1}{\psi''(s_t)}u'(c_t^u) & \text{if } j = 0 \\ -\frac{1}{\psi''(s_t)}u'(c_{t+j}^u)\beta^j \prod_{i=1}^j (1 - s_{t+i}) & \text{if } j \geq 1 \end{cases}\quad (40)$$

Similarly:

$$\frac{\partial s_t}{\partial w_{t+j}} = \begin{cases} \frac{1}{\psi''(s_t)}v'(c_t^e) & \text{if } j = 0 \\ \frac{1}{\psi''(s_t)}v'(c_{t+j}^e)\beta^j \prod_{i=1}^j (1 - s_{t+i}) & \text{if } j \geq 1 \end{cases}\quad (41)$$

and

$$\frac{\partial s_t}{\partial a_{t+j}} = \begin{cases} \frac{1}{\psi''(s_t)}[v'(c_t^e) - u'(c_t^u)] & \text{if } j = 0 \\ \frac{1}{\psi''(s_t)}[v'(c_{t+j}^e) - u'(c_{t+j}^u)]\beta^j \prod_{i=1}^j (1 - s_{t+i}) & \text{if } j \geq 1 \end{cases}\quad (42)$$

Combining the derivatives in (40), (41), and (42) for any  $j \geq 0$  leads to (11), the equation in the Lemma. Q.E.D.

**Lemma 2.** If the worker is employed at date  $t$ , then the Euler equation is

$$v'(c_t^e) = \begin{cases} \beta v'(c_{t+1}^e) & \text{if } A_t > L \\ v'(a_t + w_t + \tau_t) & \text{if } A_t = L \end{cases}\quad (43)$$

whereas the Euler condition for an unemployed agent is

$$u'(c_t^u) = \begin{cases} \beta s_{t+1}v'(c_{t+1}^e) + \beta(1 - s_{t+1})u'(c_{t+1}^u) & \text{if } A_t > L \\ u'(a_t + b_t) & \text{if } A_t = L \end{cases}\quad (44)$$

By assumption, the borrowing limit is not reached in any period between  $t$  and  $t + j$ . Therefore, recursively substituting the Euler equations, the following relationships are obtained:

$$v'(c_t^e) = \beta^j v'(c_{t+j}^e), \quad j \geq 0, \quad (45)$$

and

$$u'(c_t^u) = \beta^j v'(c_{t+j}^e) \left( 1 - \prod_{i=1}^j (1 - s_{t+i}) \right) + \beta^j u'(c_{t+j}^u) \prod_{i=1}^j (1 - s_{t+i}), \quad j \geq 1. \quad (46)$$

Subtract (46) from (45) to obtain:

$$v'(c_t^e) - u'(c_t^u) = (v'(c_{t+j}^e) - u'(c_{t+j}^u)) \beta^j \prod_{i=1}^j (1 - s_{t+i}), \quad j \geq 1. \quad (47)$$

From (42), the left hand side equals  $\psi''(s_t) \frac{\partial s_t}{\partial a_t}$  and the right hand side equals  $\psi''(s_t) \frac{\partial s_t}{\partial a_{t+j}}$ . Therefore, canceling out the term  $\psi''(s_t)$  on both sides, the expression in (12) is obtained:

$$\frac{\partial s_t}{\partial a_t} = \frac{\partial s_t}{\partial a_{t+j}}, \quad j \geq 1. \quad (48)$$

To derive the expression for  $w$  start from (41) for  $j \geq 1$  and use the recursive Euler equation relationship for the employed state (45):

$$\begin{aligned} \frac{\partial s_t}{\partial w_{t+j}} &= \frac{1}{\psi''(s_t)} v'(c_{t+j}^e) \beta^j \prod_{i=1}^j (1 - s_{t+i}) \\ &= \frac{1}{\psi''(s_t)} v'(c_t^e) \prod_{i=1}^j (1 - s_{t+i}) \\ &= \frac{\partial s_t}{\partial w_t} \prod_{i=1}^j (1 - s_{t+i}), \quad j \geq 1. \end{aligned} \quad (49)$$

This is (12), the second equation in the Lemma. Q.E.D.

**Proposition 1.** Start from the decomposition in (15), which is repeated here for convenience:

$$\frac{\partial s_0}{\partial b_j} = \frac{\partial s_0}{\partial a_0} - \frac{\partial s_0}{\partial w_0} \prod_{i=1}^j (1 - s_i), \quad j \geq 1. \quad (50)$$

Sum this equation over  $j = 1, \dots, B_1 - 1$ . Then add (11) evaluated at  $t = j = 0$  to this sum to obtain  $\frac{\partial s_0}{\partial \bar{b}_1}$  as defined in (17):

$$\begin{aligned}
\frac{\partial s_0}{\partial \bar{b}_1} &= B_1 \frac{\partial s_0}{\partial a_0} - \frac{\partial s_0}{\partial w_0} \left( 1 + \sum_{j=1}^{B_1-1} \prod_{i=1}^j (1 - s_i) \right) \\
&= B_1 \frac{\partial s_0}{\partial a_0} - \frac{\partial s_0}{\partial w_0} \frac{1}{(1 - s_0)} \left( (1 - s_0) + \sum_{j=1}^{B_1-1} \prod_{i=0}^j (1 - s_i) \right) \\
&= B_1 \frac{\partial s_0}{\partial a_0} - \frac{\partial s_0}{\partial w_0} \frac{1}{(1 - s_0)} \left( \sum_{j=0}^{B_1-1} \prod_{i=0}^j (1 - s_i) \right) \\
&= B_1 \frac{\partial s_0}{\partial a_0} - D_1 \frac{1}{1 - s_0} \frac{\partial s_0}{\partial w_0}
\end{aligned} \tag{51}$$

Sum equation (50) over  $j = B_1, \dots, B_1 + B_2 - 1$  to obtain  $\frac{\partial s_0}{\partial \bar{b}_2}$  as defined in (18):

$$\begin{aligned}
\frac{\partial s_0}{\partial \bar{b}_2} &= B_2 \frac{\partial s_0}{\partial a_0} - \frac{\partial s_0}{\partial w_0} \sum_{j=B_1}^{B_1+B_2-1} \prod_{i=1}^j (1 - s_i) \\
&= B_2 \frac{\partial s_0}{\partial a_0} - \frac{\partial s_0}{\partial w_0} \frac{1}{(1 - s_0)} \sum_{j=B_1}^{B_1+B_2-1} \prod_{i=0}^j (1 - s_i) \\
&= B_2 \frac{\partial s_0}{\partial a_0} - D_2 \frac{1}{1 - s_0} \frac{\partial s_0}{\partial w_0}
\end{aligned} \tag{52}$$

The two equations (51) and (52) can be collected in matrix form as follows:

$$\begin{bmatrix} \frac{\partial s_0}{\partial \bar{b}_1} \\ \frac{\partial s_0}{\partial \bar{b}_2} \end{bmatrix} = \begin{bmatrix} B_1 & -D_1 \\ B_2 & -D_2 \end{bmatrix} \begin{bmatrix} \frac{\partial s_0}{\partial a_0} \\ \frac{1}{1-s_0} \frac{\partial s_0}{\partial w_0} \end{bmatrix} \tag{53}$$

Because, by assumption,  $\frac{D_1}{B_1} \neq \frac{D_2}{B_2}$  the matrix admits an inverse. Pre-multiplying both sides of the equation by this inverse produces:

$$\begin{bmatrix} \frac{\partial s_0}{\partial a_0} \\ \frac{1}{1-s_0} \frac{\partial s_0}{\partial w_0} \end{bmatrix} = \frac{1}{B_2 D_1 - B_1 D_2} \begin{bmatrix} -D_2 & D_1 \\ -B_2 & B_1 \end{bmatrix} \begin{bmatrix} \frac{\partial s_0}{\partial \bar{b}_1} \\ \frac{\partial s_0}{\partial \bar{b}_2} \end{bmatrix} \tag{54}$$

Therefore,

$$\begin{aligned}
\frac{\partial s_0}{\partial a_0} &= \frac{1}{B_2 D_1 - B_1 D_2} \left( D_1 \frac{\partial s_0}{\partial \bar{b}_2} - D_2 \frac{\partial s_0}{\partial \bar{b}_1} \right) \\
\frac{\partial s_0}{\partial w_0} &= \frac{1 - s_0}{B_2 D_1 - B_1 D_2} \left( B_1 \frac{\partial s_0}{\partial \bar{b}_2} - B_2 \frac{\partial s_0}{\partial \bar{b}_1} \right).
\end{aligned} \tag{55}$$

Finally, substituting these results into (51) and (52) yields the expressions in (19) in the Proposition. Q.E.D.

**Lemma 3.** Take the total differential of  $J_0(\bar{b}_1, \bar{b}_2, \tau)$  with respect to infinitesimal changes  $d\bar{b}_1$ ,  $d\bar{b}_2$ , and  $d\tau$ . Such a policy change increases the utility of a representative worker if and only if

$$dJ_0(\bar{b}_1, \bar{b}_2, \tau) = \left. \frac{\partial J_0(\bar{b}_1, \bar{b}_2, \tau)}{\partial b} \right|_{B_1} d\bar{b}_1 + \left. \frac{\partial J_0(\bar{b}_1, \bar{b}_2, \tau)}{\partial b} \right|_{B_2} d\bar{b}_2 - \left. \frac{\partial J_0(\bar{b}_1, \bar{b}_2, \tau)}{\partial w} \right|_T d\tau \geq 0 \quad (56)$$

Notice for later use that  $\tau$  may, in principle, respond to changes in  $\bar{b}_1$  and  $\bar{b}_2$ , so that

$$d\tau = \frac{d\tau}{d\bar{b}_1} d\bar{b}_1 + \frac{d\tau}{d\bar{b}_2} d\bar{b}_2. \quad (57)$$

We need to calculate the three terms in (56). Recall that  $J_0(\bar{b}_1, \bar{b}_2, \tau) = s_0 V_0(\bar{b}_1, \bar{b}_2, \tau) + (1 - s_0) U_0(\bar{b}_1, \bar{b}_2, \tau) - \psi(s_0)$ . Therefore, the first and second terms are

$$\begin{aligned} \left. \frac{\partial J_0(\bar{b}_1, \bar{b}_2, \tau)}{\partial b} \right|_{B_1} &= (1 - s_0) \left. \frac{\partial U_0(\bar{b}_1, \bar{b}_2, \tau)}{\partial b} \right|_{B_1} \\ &= -(1 - s_0) \psi''(s_0) \frac{\partial s_0}{\partial \bar{b}_1} \\ &= -(1 - s_0) \psi''(s_0) \left( \left. \frac{\partial s_0}{\partial a} \right|_{B_1} - \left. \frac{\partial s_0}{\partial w} \right|_{B_1} \right) \end{aligned} \quad (58)$$

and

$$\begin{aligned} \left. \frac{\partial J_0(\bar{b}_1, \bar{b}_2, \tau)}{\partial b} \right|_{B_2} &= (1 - s_0) \left. \frac{\partial U_0(\bar{b}_1, \bar{b}_2, \tau)}{\partial b} \right|_{B_2} \\ &= -(1 - s_0) \psi''(s_0) \frac{\partial s_0}{\partial \bar{b}_2} \\ &= -(1 - s_0) \psi''(s_0) \left( \left. \frac{\partial s_0}{\partial a} \right|_{B_2} - \left. \frac{\partial s_0}{\partial w} \right|_{B_2} \right) \end{aligned} \quad (59)$$

Also, the third term can be calculated as

$$\begin{aligned}
\left. \frac{\partial J_0(\bar{b}_1, \bar{b}_2, \tau)}{\partial w} \right|_T &= s_0 \left. \frac{\partial V_0(\bar{b}_1, \bar{b}_2, \tau)}{\partial w} \right|_T + (1 - s_0) \left. \frac{\partial U_0(\bar{b}_1, \bar{b}_2, \tau)}{\partial w} \right|_T \\
&= s_0 \sum_{t=0}^{T-1} \beta^t v'(c_t^e) + (1 - s_0) \sum_{t=1}^{T-1} (1 - \prod_{i=1}^t (1 - s_i)) \beta^t v'(c_t^e) \\
&= s_0 v'(c_0^e) + s_0 \sum_{t=1}^{T-1} \beta^t v'(c_t^e) + (1 - s_0) \sum_{t=1}^{T-1} \beta^t v'(c_t^e) - (1 - s_0) \sum_{t=1}^{T-1} \prod_{i=1}^t (1 - s_i) \beta^t v'(c_t^e) \\
&= s_0 v'(c_0^e) + \sum_{t=1}^{T-1} \beta^t v'(c_t^e) - (1 - s_0) \sum_{t=1}^{T-1} \prod_{i=1}^t (1 - s_i) \beta^t v'(c_t^e) \\
&= s_0 v'(c_0^e) + (T - 1) v'(c_0^e) - (1 - s_0) \sum_{t=1}^{T-1} \prod_{i=1}^t (1 - s_i) \beta^t v'(c_t^e) \\
&= \psi''(s_0) \left( T \frac{\partial s_0}{\partial w_0} - (1 - s_0) \frac{\partial s_0}{\partial w_0} - (1 - s_0) \sum_{t=1}^{T-1} \frac{\partial s_0}{\partial w_t} \right) \\
&= \psi''(s_0) \left( T \frac{\partial s_0}{\partial w_0} - (1 - s_0) \frac{\partial s_0}{\partial w_0} - (1 - s_0) \frac{\partial s_0}{\partial w_0} \sum_{t=1}^{T-1} \prod_{i=1}^t (1 - s_i) \right) \\
&= \psi''(s_0) \frac{\partial s_0}{\partial w_0} \left( T - (1 - s_0) - \sum_{t=1}^{T-1} \prod_{i=0}^t (1 - s_i) \right) \\
&= \psi''(s_0) \frac{\partial s_0}{\partial w_0} (T - (1 - s_0) - (D - (1 - s_0))) \\
&= \psi''(s_0) \frac{\partial s_0}{\partial w_0} (T - D) \\
&= (1 - s_0) \psi''(s_0) \frac{T - D}{D_k} \frac{\partial s_0}{\partial w} \Big|_{B_k}, \quad k = 1, 2 \tag{60}
\end{aligned}$$

The equality in the first line uses the relationship  $J_0 = s_0 V_0 + (1 - s_0) U_0$ . The second line uses the derivatives calculated in (38). Notice that  $\frac{\partial U_0}{\partial w_0} = 0$  and that therefore the summation of the last term starts at 1. The third and fourth lines are algebraic manipulations. The fifth line uses the Euler equation for the employed state. The sixth line uses the derivatives in (41) specialized to  $t = 0$ . The seventh line uses the relationship between  $\frac{\partial s_0}{\partial w_t}$  and  $\frac{\partial s_0}{\partial w_0}$ . The next three lines are algebraic manipulations and the last line uses the relationship  $\frac{\partial s_0}{\partial w} \Big|_{B_k} = \frac{D_k}{1 - s_0} \frac{\partial s_0}{\partial w_0}$ .

Substitute all the above results into (56) to obtain that  $dJ_0 \geq 0$  if and only if

$$-(LIQ_1 - MH_1) d\bar{b}_1 - (LIQ_2 - MH_2) d\bar{b}_2 - \frac{T - D}{D_1} MH_1 \frac{d\tau}{d\bar{b}_1} d\bar{b}_1 - \frac{T - D}{D_2} MH_2 \frac{d\tau}{d\bar{b}_2} d\bar{b}_2 \geq 0 \tag{61}$$

After combining the first with the third term and the second with the fourth term, the expression in the Lemma is obtained. Q.E.D.

**Lemma 4.** On the expense side, raising  $\bar{b}_k$  increments the cost of providing insurance according to

$$\begin{aligned} \frac{dC(\mathbf{b})}{d\bar{b}_k} &= D_k + b_1 \frac{dD_1}{d\bar{b}_k} + b_2 \frac{dD_2}{d\bar{b}_k}, \quad k = 1, 2 \\ &= D_k \left( 1 + \varepsilon_{D_k, \bar{b}_k} + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \varepsilon_{D_{k'}, \bar{b}_k} \right), \quad k' \neq k, k = 1, 2. \end{aligned} \quad (62)$$

Revenue is  $Rev(\mathbf{b}) = \tau \sum_{t=0}^{T-1} (1 - S_t) = \tau(T - D)$ , so that

$$\begin{aligned} \frac{dRev(\mathbf{b})}{d\bar{b}_k} &= (T - D) \frac{d\tau}{d\bar{b}_k} + \tau \frac{d(T - D)}{d\bar{b}_k} \\ &= (T - D) \frac{d\tau}{d\bar{b}_k} - \frac{\tau D}{\bar{b}_k} \varepsilon_{D, \bar{b}_k}, \quad k = 1, 2. \end{aligned} \quad (63)$$

If the increase in expenses is to be balanced by an increase in revenue, then

$$\frac{dRev(\mathbf{b})}{d\bar{b}_j} = \frac{dC(\mathbf{b})}{d\bar{b}_j}. \quad (64)$$

After combining the results for the expense side and the revenue side, and solving for  $\frac{d\tau}{d\bar{b}_k}$ ,

$$\frac{d\tau}{d\bar{b}_k} = \frac{1}{T - D} \left[ D_k \left( 1 + \varepsilon_{D_k, \bar{b}_k} + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \varepsilon_{D_{k'}, \bar{b}_k} \right) + \frac{\tau D}{\bar{b}_k} \varepsilon_{D, \bar{b}_k} \right], \quad k' \neq k, k = 1, 2. \quad (65)$$

If the budget was originally balanced, so that  $\tau(T - D) = \bar{b}_1 D_1 + \bar{b}_2 D_2$ , then this equation becomes

$$\frac{d\tau}{d\bar{b}_k} = \frac{D_k}{T - D} \left[ 1 + \varepsilon_{D_k, \bar{b}_k} + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \varepsilon_{D_{k'}, \bar{b}_k} + \frac{D}{T - D} \left( 1 + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \right) \varepsilon_{D, \bar{b}_k} \right], \quad k' \neq k, k = 1, 2. \quad (66)$$

This is the expression in the Lemma. Q.E.D.

**Proposition 2.** From Lemma 3,  $\frac{dJ_0}{d\bar{b}_k} \geq 0$  as long as

$$- LIQ_k + MH_k \left( 1 - \frac{T - D}{D_k} \frac{d\tau}{d\bar{b}_k} \right) \geq 0, \quad (67)$$

Using the result in Lemma 4 to substitute for  $\frac{d\tau}{d\bar{b}_k}$ , this expression becomes:

$$- LIQ_k - MH_k \left[ \varepsilon_{D_k, \bar{b}_k} + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \varepsilon_{D_{k'}, \bar{b}_k} + \frac{D}{T - D} \left( 1 + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \right) \varepsilon_{D, \bar{b}_k} \right] \geq 0 \quad (68)$$



Equivalently,

$$-\frac{LIQ_k}{MH_k} \geq \varepsilon_{D_k, \bar{b}_k} + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \varepsilon_{D_{k'}, \bar{b}_k} + \frac{D}{T-D} \left( 1 + \frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k} \right) \varepsilon_{D, \bar{b}_k}. \quad (69)$$

At an interior optimum,  $\frac{dJ_0}{d\bar{b}_k} = 0$ , which implies that this weak inequality becomes an equality. Q.E.D.