Lobbying, Inside and Out:
How Special Interest Groups Influence Policy Choices*

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Abstract

Scholars have long recognized two classes of special interest group (SIG) expenditures: inside lobbying, which is intended to influence the content of a bill; and outside lobbying, which is intended to influence the likelihood a bill is enacted into law. This paper juxtaposes both lobbying activities within a single model. Policy choices are a function of the decision-maker’s assessment of SIGs’ willingness to engage in outside lobbying. Importantly, inside lobbying expenditures do not always reflect SIGs’ outside lobbying capacities and therefore cannot adequately measure SIG influence. Consequently, empirical studies of SIG influence which exclusively consider inside lobbying expenditures—as nearly all existing tests do—are likely to produce spurious results. The paper highlights that strong SIG influence is consistent with a small effect of inside lobbying expenditures on policy choice.

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Introduction

The popular wisdom, as expressed in copious news articles and polls, holds that the political process is rigged in favor of Special Interest Groups (SIGs) due to the presence of money in politics. This popular wisdom has so far received little empirical validation. Politicians do not seem to accord too much importance to contributions (i.e., monetary transfers) or lobbying expenditures (i.e., transfers of information). Out of the 25 most powerful SIGs in 1999 according to members of Congress and their staff, only 12 belong to the top 25 for contributions and only 4 to the top 25 for lobbying expenditures. Following multiple studies documenting a small effect of SIG’s spending on policymakers’ decisions (for a review, see Ansolabehere et al., 2003), the new scholarly consensus is that money does not buy policies (Azari, 2015).

This new consensus, however, relies on analyses focusing exclusively on one type of SIGs’ expenditures: inside lobbying expenditures—contributions and lobbying spending—meant generally to shape the content of a bill. While inside lobbying expenditures are undoubtedly important, SIGs have other means to influence policies. One prominent example is outside lobbying—issue advocacy advertising or grass-roots mobilization—intended to affect the likelihood a bill is enacted into law.

In this paper, I incorporate both inside and outside lobbying activities within a single game-theoretic framework. I show that empirical tests of SIG influence—defined as SIGs’ ability to bias political decisions in their favor—which consider exclusively inside lobbying expenditures suffer from attenuation bias and can yield wrongly signed estimates. Nonetheless, SIGs have strong influence on policy choices. The popular wisdom is not invalidated by the small effect of inside lobbying expenditures on policy choice. This paper suggests that to recover unbiased estimate of the extent of SIG influence, empirical researchers would do better to use outside lobbying expenditures.

I study a set-up with three players: a decision-maker (to whom I reserve the pronoun ‘she’), a pro-change SIG who, like the decision-maker, is favorable to policy change, and an anti-change SIG who prefers the status quo. The decision-maker chooses the content of a bill (i.e, the extent of the policy change), whereas the SIGs decide how much inside lobbying expenditures to incur and whether to engage in outside lobbying activities. Lobbying activities are costly and this cost

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(assumed to be the same for inside and outside lobbying) depends on the SIG’s strength, which is its private information. A weak SIG faces a high lobbying cost, whereas this cost is low for a strong SIG.

In this model, an SIG can use inside lobbying to signal its strength. As such, inside lobbying expenditures influence policy choices only when SIGs play a separating strategy (i.e., the two types send different signals). Outside lobbying expenditures serve a very different purpose. When it engages in outside lobbying, an anti-change SIG reduces the probability that the decision-maker’s bill is enacted into law. This probability is 1 absent outside lobbying activities and strictly less than one, otherwise. It can even be null if the decision-maker does not pay a cost to defend her proposal. The decision-maker’s cost of defending her proposal depends on whether the pro-change SIG engages in outside lobbying and is strictly lower when it does. Only a strong pro-change SIG, however, is capable of bearing the cost of outside lobbying activities.

I find that when the anti-change SIG plays a separating strategy, inside lobbying expenditures are perfectly correlated with influence. Only a strong anti-change SIG incurs positive inside lobbying expenditures, the decision-maker compromises with a strong anti-change SIG by proposing a small policy change, and no type engages in outside lobbying. A key finding of this paper is that even though the single-crossing condition holds (a strong SIG faces a lower lobbying cost), a separating equilibrium exists only if a strong anti-change SIG’s lobbying cost is intermediary.

For low lobbying cost, a separating equilibrium does not exist because the decision-maker does not want to compromise. The decision-maker would need to propose a very moderate a policy change to avoid outside lobbying activities by a strong SIG. She then prefers to propose a large policy change even if it induces outside lobbying activity by the anti-change SIG. For relatively high lobbying cost, a separating equilibrium does not exist because a strong anti-change SIG does not have sufficient incentive to reveal its type. To understand this result, notice that in a separating assessment, a weak anti-change SIG’s benefit of imitating a strong type is high since it always obtains a bill closer to the status quo. In contrast, the benefit of revealing its strength is relatively low for a strong SIG. While it faces a bill further away from the status quo by pretending to be weak, it has the option to engage in outside lobbying and reduce the probability the bill is enacted into law. Consequently, a weak type’s benefit from imitation is always greater than a strong type’s benefit from differentiation. Consequently, a strong anti-change SIG is not willing to bear the
necessary lobbying expenditures to credibly reveal its type when its lobbying cost is relatively close to a weak SIG’s.

When a strong anti-change SIG’s lobbying cost is low or (relatively) high, only pooling equilibria exist (i.e., strong and weak types play the same inside lobbying strategy). These equilibria differ only in the level of inside lobbying expenditures incurred by the anti-change SIG. But these expenditures have no effect on the decision-maker’s policy choice, which depends only on her prior about the SIG’s strength. Consequently, empirical tests of SIG influence using exclusively inside lobbying expenditures as independent variable are likely to suffer from attenuation bias. This attenuation bias is still present even after controlling for potential lobbying costs and focusing on parameter values such as a separating equilibrium exist. Indeed, the separating equilibrium is not necessarily unique so empirical researchers are confronted with an equilibrium selection problem that inside lobbying expenditures cannot help to resolve.

Outside lobbying expenditures, on the other hand, allow researchers to recover an unbiased estimate of SIG influence. An anti-change SIG engages in outside lobbying activities only when it fails to bias the decision-maker’s proposal at the policy stage. Consequently, outside lobbying expenditures measure the extent of anti-change SIG’s lack of influence. On the other hand, they reveal little about how anti-change SIGs influence policies. As such, this paper highlights the difficulty to obtain a complete understanding of anti-change SIG influence.

The pro-change SIG faces a different trade-off than the anti-change SIG. When it reveals its strength, a strong pro-change SIG induces the decision-maker to choose a more radical policy change (i.e., a bill further away from the status quo). However, in exchange, a strong pro-change SIG must subsidize the cost of defending the decision-maker’s proposal when the anti-change SIG engages in outside lobbying. A strong pro-change SIG has then no incentive to incur inside lobbying expenditures to reveal that it is willing to engage in costly outside lobbying activities. In a separating equilibrium, only a weak pro-change SIG uses inside lobbying expenditures to “plead poverty.” When it comes to a pro-change SIG, inside lobbying expenditures are inversely correlated with influence. Conditioning on pro-change SIGs playing a separating strategy, empirical analyses using exclusively inside lobbying expenditures are likely to yield wrongly signed estimates of SIG influence. Importantly, a strong pro-change SIG does not obtain a favorable policy at low cost. Total expenditures (inside and outside lobbying combined) are always higher for the strong type.
As for the anti-change SIG, a separating equilibrium does not exist for all parameter values. Whenever a strong pro-change SIG’s lobbying cost is high, the cost of helping to defend the decision-maker’s proposal is too high compared to the policy gain and a strong type has no incentive to reveal its strength. Consequently, for a non-trivial range of parameter values, only pooling equilibria (differing only in their uninformative level of inside lobbying expenditures) exist. As pooling equilibria also exist when a separating equilibrium does (in some circumstances), an empirical researcher faces the same problem of equilibrium selection and the same risk of attenuation bias as in the case of anti-change SIG.

For a pro-change SIG, inside lobbying expenditures are a poor proxy of SIG influence. Outside lobbying expenditures, on the other hand, yield unbiased estimates of influence. As the decision-maker chooses a more radical policy change only if she expects a strong pro-change SIG’s help, outside lobbying activities thus identify the extent of pro-change SIG and how they bias policies in their favor.

I conclude this introduction by connecting this paper to the most closely related theoretical literature. Copious studies investigate SIG influence under the assumption that contributions are a form of bribery (e.g., Denzau and Munger, 1986; Grossman and Helpman, 1996 and 2001; Besley and Coate, 2001). Other papers suppose that SIG money buys access in order to transmit information about the impact of a proposed policy change on constituents’ welfare (e.g., Potters and Van Widen, 1992; Austen-Smith, 1995; Ball, 1995; Lohmann, 1995; Cotton, 2011). The present paper complements this literature by interpreting inside lobbying expenditures as a signal of SIGs’ willingness to participate in costly activities which influence the fate of a bill.

Some papers assume that SIGs use some sort of outside lobbying activities to influence political decisions. Yu (2005) studies a model where SIGs can raise the salience of an issue before engaging in quid pro quo contributions. Kollman (1998) supposes that outside lobbying activities can change a policy-maker’s legislative agenda. Bombadini and Trebbi (2011) assume that firms can use money or promise votes by mobilizing their employees in exchange for public subsidies. They find that only intermediary-sized firms use money as small firms find it too costly to enter politics and large

3See also Groll and Prummer (2015) who study informative lobbying in a political network.
4Gordon and Hafer (2005, 2007) also consider a set-up in which inside lobbying expenditures signal firms’ willingness to contest an agency’s regulatory decision. The SIG’s choice to effectively contest regulation, however, is left unmodeled.
firms can promise votes to get their preferred policy. In contrast, I assume that outside lobbying activities occur after the decision-maker’s policy choice and affect the likelihood her bill is enacted into law. Outside lobbying (by the anti-change SIG) can thus be interpreted as a threat.

Several works analyze how threats affect the political process. Ellman and Wantchekon (2000) study how the threat of civil war biases electoral platforms in favor of the party backed by potential rebels. Scartascini and Tommasi (2012) adapt this set-up to legislative bargaining. Wolton (2015) investigates how threats by the rich induce a governing party to compromise on taxation and shows that the presence of opposition party can be Pareto improving for political actors and interest groups alike. Dal Bó and Di Tella (2003) and Dal Bó et al. (2006) analyze how threats increase the effectiveness of bribes. Dahm and Porteiro (2008) suppose that an SIG can perform a test to reveal information about an ex-ante unknown state of the world prior to engaging in political pressure which affects the probability a bill is enacted into law. They show that the SIG generally prefers a public test (where results are observed by all players) to a private test (where only the SIG observes the result), but do not consider how political pressure affects estimates of SIG influence. None of these papers assumes that SIGs can use money to transmit private information and so have little to say about how outside lobbying activities affect the informativeness of inside lobbying expenditures.

Evidence on inside and outside lobbying

Most empirical studies of SIG influence consider how campaign contributions affect politicians’ decisions, especially a legislator’s vote. A common finding in this literature is that the effect of contributions is statistically insignificant when legislators’ ideology is controlled for (see Ansolabehere et al., 2003 Table 1). This approach, however, suffers from three serious flaws.

First, bills scheduled for a vote are not exogenous. They are result of equilibrium plays anticipating legislators’ and SIGs’ actions at the voting stage. It is thus somewhat unsurprising that contributions have little effect so late in the political game. Empirical papers considering policies rather than votes have generally found a small, but positive impact of contributions. Examples include Goldberg and Maggi (1999) who find weak support for Grossman and Helpman’s (1994) theory of trade protection, Bombardini and Trebbi (2011) for firm subsidies, and (in a related fashion) Hall and Wayman (1990) for legislators’ effort at the committee stage.
Second, contributions only represent a small portion of SIGs’ inside lobbying expenditures (20% during the 2011-12 electoral cycle as documented by the Center for Responsive Politics). SIGs invest substantially more in the transmission of information (i.e., hiring lobbyists). Empirically, lobbying seems to have a positive effect on the content of bills when it comes to academic earmarks (de Figueiredo and Silverman, 2006) and corporate taxes (Richter et al., 2009), or on the success of energy policy proposals (Kang, 2015).

Third, most empirical studies consider exclusively inside lobbying expenditures, assuming implicitly that other SIG activities are of little consequences for their results. As this paper shows, this “all else equals” claim needs to be qualified. Empirical researchers need to consider SIGs’ outside lobbying activities to recover unbiased estimates of influence. As such, this paper joins a long scholarly tradition stressing the importance of outside lobbying activities (see Blaisdell, 1957; Wright, 1996: 90; Kollman, 1998: 103; Hojnacki and Kimball, 1999: 1005-6; Baumgartner et al., 2009: 150-7). Due to the absence of empirical evaluation of outside lobbying, there is, however, no consensus on the purpose(s) of these activities. Nonetheless, it is well documented that SIGs have used issue advocacy advertising to affect the fate of prominent legislative reforms—for example, Clinton’s 1993 health care reform (West et al., 1996; Goldstein, 1999), the 1998 Senate tobacco bill (Jamieson, 2000; Derthick, 2012), Obama’s 2010 Affordable Care Act (Hall and Anderson, 2012; LaPira, 2012). Further, as documented by Lord (2000), lobbyists and members of Congress report that inside lobbying expenditures are more effective to shape the content of a bill, whereas outside lobbying (constituency building) has greater impact on the legislative success of a policy proposal. Using this (partial) evidence, this paper provides a theoretical framework to understand the impact of outside lobbying activities (understood in what follows as issue advocacy advertising) on political

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5 Falk et al. (2006) estimate that issue advocacy advertising amounted to more than $400 million in the Washington DC media market alone during the 108th Congress. In comparison, SIGs contributed approximately $570m to Members of Congress (excluding presidential-candidate John Kerry), and spent $4bn on lobbying during the 2003-2004 electoral cycle (source: Center for Responsive Politics).

6 An exception is Hall and Reynold (2012) who study the targets of issue advocacy advertising, but do not analyze the impact of this type of advertising on voting decisions.

7 Commenting on the success of the Harry and Louise campaign in the debate on Clinton’s Health Care Reform in 1993, Bill McInturff, who helped in the campaign, explains, “In terms of the questions raised about the “public policy process,” if the White House cannot build majority support faced with “soft” advertising that raised simple and fundamental questions, it suggests to our firm that we have materially made a contribution to the process by not allowing such a substantial piece of legislation to pass without a full airing of its consequences.” (cited in Brodie, 2001).
decisions when they are meant to affect the likelihood a bill is enacted into law.\footnote{Notice that this paper does not consider important outside lobbying activities such as grass-roots mobilization, which, for example, played a role in President Obama’s decision to reject the Keystone XL oil pipeline (Davenport, 2015).}

**The model**

I study a one-period three-player game with a decision-maker (superscript $D$), a pro-change SIG ($P$), and an anti-change SIG ($A$). The decision-maker and the pro-change (anti-change) SIG have similar (opposite) policy preferences. The game has three parts. In the first stage, SIGs observe their strength and decide whether to reveal it to the decision-maker via their inside lobbying strategy. In the second stage, the decision-maker decides the content of a bill $b \in [0, 1]$, where 0 represents the status quo and 1 the preferred policy of the decision-making and pro-change SIG. In the third part (‘outside lobbying’ subgame), SIGs decide whether to engage in outside lobbying ($l^J_0 \in \{0, 1\}$ for $J \in \{A, P\}$).

Outside lobbying activities have an impact on the outcome of the game denoted $y \in \{0, b\}$. When the anti-change SIG does not engage in outside lobbying ($l^A_0 = 0$), the decision-maker’s bill is always enacted into law: $y = b$. Otherwise ($l^A_0 = 1$), the outcome depends on the decision-maker and the pro-change SIG’s choices. The decision-maker chooses a response $d \in \{0, 1, 2\}$ to the anti-change SIG’s outside lobbying activities. When the decision-maker chooses $d = 0$, she backs down and the status quo is always helped: $y = 0$. When the decision-maker chooses $d = 2$, she defends the bill on her own and the bill is passed ($y = b$) with probability $p$; with probability $1 - p$, the status quo is upheld ($y = 0$). Finally, when the decision-maker chooses $d = 1$, she asks for the pro-change SIG’s support and the outcome then depends on the pro-change SIG’s decision whether to engage in outside lobbying. If the pro-change SIG engages in outside lobbying ($l^P_0 = 1$), the bill is passed with probability $p$; if not ($l^P_0 = 0$), the bill is not enacted into law (i.e., $y = 0$ with probability 1).

Outside lobbying is costly for all players. The decision-maker cost of a response $d$ is: $\frac{k}{2} \times d$. Importantly, the decision-maker faces a lower cost when she asks for support ($d = 1$) than when she defends the bill on her own ($d = 2$). This cost is common knowledge. On the other hand, an SIG’s cost of outside lobbying is its private information and depends on its strength (type) $\tau \in \{s, w\}$,
where $s$ denotes strong and $w$ denotes weak. A strong SIG $J \in \{A, P\}$ faces a lower cost of outside lobbying activities: $c_w^J > c_s^J$. It is common knowledge that the SIG’s types are uncorrelated and the proportion of strong SIG $J \in \{A, P\}$ is $Pr(\tau^J = s) = q^J \in [0, 1)$.

In the first stage, SIGs can reveal their type by engaging in inside lobbying. SIG $J \in \{A, P\}$’s inside lobbying activities take the form of a costless message $m \in \{s, w\}$ and costly expenditures $l_i^J \geq 0$. Denote $\zeta^J := (m, l_i^J)$ the signal of SIG $J$. The cost of inside lobbying expenditures depends on the SIG’s strength and, for ease of exposition, is equal to the cost of outside lobbying activities: $c_\tau^J, J \in \{A, P\}, \tau \in \{s, w\}$. Henceforth, I refer to $c_\tau^J$ as ‘the lobbying cost.’ Observe that since types are drawn independently, the anti-change SIG’s signal reveals no information about the pro-change SIG’s strength and vice versa.

As noted above, the decision-maker’s preferred outcome ($y$) is 1. Incorporating the cost of responding to the anti-change SIG’s outside lobbying activities, her utility function can be expressed as:

$$u^D(y, d) = y - \frac{kd}{2} \quad (1)$$

The pro-change SIG’s preferred outcome is the same as the decision-maker’s. Its utility function also includes the cost of both inside lobbying expenditures ($l_i^P \geq 0$) and outside lobbying activities ($l_o^P \in \{0, 1\}$) and thus assumes the following form:

$$u^P(y, l_i^P, l_o^P; \tau) = y - c_\tau^P(l_o^P + l_i^P), \; \tau \in \{s, w\} \quad (2)$$

The anti-change SIG prefers the status quo ($y = 0$), any change imposes a payoff loss. Adding the cost of inside ($l_i^A \geq 0$) and outside lobbying ($l_o^A \in \{0, 1\}$), its utility function is:

$$u^A(y, l_i^A, l_o^A; \tau) = -y - c_\tau^A(l_o^A + l_i^A), \; \tau \in \{s, w\} \quad (3)$$

To summarize the timing of the game is:

1. Nature draws SIGs’ types independently: $\tau^J \in \{s, w\}, \; J \in \{A, P\}$;

2. Both SIGs privately observe their type and send simultaneously a signal: $\zeta^J = (m, l_i^J) \in \{s, w\} \times \mathbb{R}_+$;
3. The decision-maker chooses the content of the bill: \( b \in [0, 1] \);

4. The anti-change SIG decides whether to engage in outside lobbying: \( l^A_o \in \{0, 1\} \);

5. The decision-maker then decides whether to back down, ask for support, or defend her proposal: \( d \in \{0, 1, 2\} \);

6. The pro-change SIG decides whether to engage in outside lobbying: \( l^P_o \in \{0, 1\} \).

**Outcome:**

1. If the anti-change SIG does not engage in outside lobbying (\( l^A_o = 0 \)): \( y = b \)

2. If the anti-change SIG engages in outside lobbying (\( l^A_o = 1 \)):
   
   \[
   \begin{cases} 
   y = b \text{ with probability } \mathbb{I}_{\{d + l^P_o \geq 2\}} (1 - p) \\
   y = 0 \text{ with probability } 1 - \mathbb{I}_{\{d + l^P_o \geq 2\}} (1 - p)
   \end{cases}
   \]

   where \( \mathbb{I}_{\{d + l^P_o \geq 2\}} \) is the indicator function equals to 1 if \( d + l^P_o \geq 2 \), and 0, otherwise.

The equilibrium concept is Perfect Bayesian Equilibrium (PBE) in pure strategies (see Supplemental Appendix A for a formal definition). A PBE requires that a) each player’s choices be sequentially rational given her belief at the time of choice and other players’ strategies; b) beliefs satisfy Bayes’ rule on the equilibrium path (are consistent with priors and equilibrium strategies). As it is common in signaling games, multiple PBE can emerge absent additional restrictions. I therefore impose the Intuitive Criterion (Cho and Kreps, 1987). To restrict the number of outcome-equivalent equilibria and facilitate the exposition, I also impose that a pro-change SIG’s signal as a function of its strength—denoted \( \zeta^P(\tau), \tau \in \{s, w\} \)—satisfies \( \zeta^P(s) \neq \zeta^P(w) \) only if its inside lobbying activity influences the decision-maker and anti-change SIG’s strategies on the equilibrium path. In what follows, the term ‘equilibrium’ refers to this class of equilibria.

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9 In Supplemental Appendix B, I show that the main results hold when I allow for mixed strategies.

10 Define the decision-maker’s policy choice \( b \) as a function of the pro-change SIG’s signal \( \zeta^P \) and the anti-change SIG’s signal \( \zeta^A \); and her response \( d \) as a function of \( \zeta^P, \zeta^A, b, l^P_o \), the content of the bill \( b \), and the anti-change SIG’s outside lobbying activities \( l^A_p \). This restriction imposes that if \( b(\zeta^P, \zeta^A) = b(\zeta'^P, \zeta^A) \) and \( d(\zeta^P, \zeta^A, b(\zeta^P, \zeta^A, l^A_o)) = d(\zeta'^P, \zeta^A, b(\zeta'^P, \zeta^A, l^A_o)) \) for all \( \zeta^P \neq \zeta'^P \) and for all \( \zeta^A \in \{s, w\} \times \mathbb{R}_+, b(\cdot) \in [0, 1], \) and \( l^A_o \in \{0, 1\} \) on the equilibrium path (i.e., the pro-change SIG’s strategy has no impact on equilibrium outcome), then the pro-change SIG plays a pooling strategy \( (\zeta^P(s) = \zeta^P(w)) \). Footnote 13 details the role played by this restriction in the analysis.
Assumptions

To limit the number of cases to be considered and ensure both SIGs’ signals play a role in the analysis, I impose some conditions on the model parameters. For the anti-change SIG, I assume that the following inequalities hold:

**Assumption 1.** A weak anti-change SIG’s lobbying cost satisfies: \( \max \left\{ 1 - p - k, \frac{k}{1-p} \right\} < \frac{c_w}{p} < 1 \)

The lower bound guarantees that the decision-maker prefers to avoid outside lobbying activities by a weak anti-change SIG. The upper bound is for exposition purposes only, and all results hold when it is relaxed.

Regarding the pro-change SIG, I impose the following conditions.

**Assumption 2.** The following inequalities hold: i. \((1 - p)\frac{c_{aw}}{p} < c_s^P < (1 - p) < c_w^P\) and ii. \(q^P \leq \frac{1}{2}\)

Point i. implies that engaging in outside lobbying is a strictly dominated strategy for a weak pro-change SIG (the inequality \((1 - p)\frac{c_{aw}}{p}\) is meant to simplify the analysis, but does not affect the main results). Point ii. is a sufficient condition such that when the pro-change SIG’s inside lobbying activities reveal no information about its strength, the decision-maker does not ask for support \((d = 1)\).

Discussion

This game shares similarities with traditional signaling games, with one important twist. In traditional signaling games, the sender sends a signal, the receiver decides what action to take after observing the signal, and the game ends. In the present set-up, the game does not end after the receiver’s (decision-maker’s) policy choice. One of the senders (the anti-change SIG) has the opportunity to act again (engage in outside lobbying) to affect the final outcome of the game. This assumption, which corresponds to the idea that outside lobbying is intended to influence the likelihood that a bill is enacted into law, is the key force behind the results below\(^{11}\)

The anti-change SIG’s outside lobbying activities in this set-up can be thought as airing ads attacking the decision-maker in a future (unmodeled) election. It can also be understood as issue

\(^{11}\)Models of signaling in the shadow of war exhibit a similar feature (Fearon, 1997; Arena, 2013). However, signaling in these models is a binary choice variable, the single-crossing condition does not hold, and there is no equivalent to a pro-change SIG. As such, the present paper is substantively and technically different.
advocacy advertising to inform the public of the consequences of the decision-maker’s proposal. In Supplemental Appendix D I show how outside lobbying activities can be understood as a war of information, in the sense of Gül and Pesendorfer (2012). The pro-change SIG’s outside lobbying activities can be understood as ads defending the decision-maker’s proposal and thus as a form of subsidy.

Another important assumption is that the cost of inside lobbying expenditures depends on an SIG’s strength: the single-crossing condition holds. Both inside and outside lobbying activities require money and an SIG’s strength captures its capacity to collect funds from its members or usual donors. As pointed out by Ainsworth (2000: 122), policy-makers have little information about SIGs’ membership and members’ willingness to become active on particular issues.

Other technical assumptions are simply for ease of exposition. The assumption that the decision-maker’s bill is always enacted into law absent outside lobbying activities can be relaxed as long as it is strictly higher than the probability of enactment when the anti-change SIG engages in outside lobbying. The same holds true for the assumption that the bill is never enacted into law when the decision-maker does not respond to the anti-change SIG’s attacks. The key insights are unchanged when players are risk-adverse or when an SIG’s strength affects differently the cost of inside and outside lobbying expenditures. Inside lobbying expenditures correspond to burning money in the present set-up, but part of it could be a transfer to the decision-maker (e.g., contributions) without affecting the main results. Imposing that SIGs can only engage in outside lobbying after the decision-maker has made her proposal is without loss of generality. Since SIGs correctly anticipate future actions, their equilibrium behavior would remain the same if they can start an outside lobbying campaign before the decision-maker decides on the content of the bill. Finally, the main conclusions hold for any finite type-space (but the analysis becomes significantly more complicated).

**Pro-change SIG influence**

I first analyze how the pro-change SIG influences policy choices. To this end, I assume that it is common knowledge that the anti-change SIG is weak: \( q^A = 0 \). Consequently, an anti-change SIG’s signal \( \zeta^A \) reveals no information and an anti-change SIG’s inside lobbying strategy can be ignored without loss of generality.
At the policy-making stage, the decision-maker faces a choice between proposing her preferred policy \( b = 1 \) or finding a compromise with the anti-change SIG. A compromise takes the form of a bill which leaves the anti-change SIG indifferent between \( l_o^A = 1 \) (engaging in outside lobbying) and \( l_o^A = 0 \). Denote \( b_w \) this ‘compromise bill.’ Simple algebra yields \( b_w := \frac{c_A}{p} < 1 \). Assumption 1 guarantees that the compromise is credible since the decision-maker is willing to defend the compromise bill whenever the anti-change SIG engages in outside lobbying.\(^{12}\) Furthermore, Assumptions 1 and 2 imply that absent any additional information about the pro-change SIG’s strength, the decision-maker always chooses the compromise bill. Proposing her preferred policy leads to an expected payoff \( 1 - p - k \) (since the anti-change SIG engages in outside lobbying and the decision-maker defends her bill alone), strictly lower than \( b_w \).

Since compromise is the decision-maker’s default option, the pro-change SIG can obtain a more favorable policy \( b = 1 \) only when it plays a separating strategy (i.e., \( \xi^P(s) \neq \xi^P(w) \)). But a strong pro-change SIG also has some incentive to hide its type so as not to pay the cost of defending the decision-maker’s proposal. The next lemma establishes under which conditions a separating equilibrium exists.

**Lemma 1.** There exists a unique \( c^P : [0, 1]^2 \to (0, (1 - p)) \) such that a separating equilibrium exists if and only if i. the compromise bill \( b_w \) satisfies \( k/2 \leq 1 - p - b_w \); and ii. a strong pro-change SIG’s lobbying cost satisfies: \( c_s^P \leq c^P(b_w, c_w^P) \).

A separating equilibrium exists when a strong pro-change SIG’s benefit from differentiation is greater than the associated cost. The benefit from differentiation is positive only if the decision-maker chooses her preferred bill \( b = 1 \) after learning the pro-change SIG is strong. The decision-maker must thus prefer the lottery induced by choosing \( b = 1 \), anticipating the anti-change SIG’s outside lobbying activities and the strong pro-change SIG’s help, to the certain payoff from proposing the compromise bill \( b = b_w \). That is, a first necessary condition is: \( b_w \leq 1 - p - k/2 \) (Condition i.)\(^{13}\) The cost from differentiation corresponds to the cost of outside lobbying activities since the

\(^{12}\)Suppose the anti-change SIG chooses \( l_o^A = 1 \), the decision-maker gets \( (1 - p)b_w - k \) when she defends her bill alone (as the bill is passed with probability \( 1 - p \) and the cost of defending it is \( k \)) and 0 otherwise. Assumption 1 guarantees \( (1 - p)b_w - k > 0 \).

\(^{13}\)When this condition is not satisfied, the decision-maker chooses \( b = b_w \), the anti-change SIG chooses \( l_o^A = 0 \) on the equilibrium path independently of the pro-change SIG’s signal. The restriction on the pro-change SIG’s equilibrium behavior then implies that the pro-change SIG plays a pooling strategy (i.e., \( \xi^P(s) = \xi^P(w) \)). Absent this restriction, the pro-change SIG might be willing to truthfully reveal its type, but this would not affect other
decision-maker asks for support \((d = 1)\) after choosing \(b = 1\). A second necessary condition is then that the lobbying cost is not too large: \(c^P_s \leq \bar{c}P(b_w, c^P_w)\) (Condition ii.).

Denote \(b^*(\zeta^A, \zeta^P)\) and \(y^*(\zeta^A, \zeta^P)\) the equilibrium bill and outcome as a function of the pro-change SIG’s signal. The next remark establishes that in a separating equilibrium, a strong pro-change SIG obtains a more favorable bill and a more favorable outcome in expectation (despite the lottery induced by the anti-change SIG’s outside lobbying activities). As such, considering (average) laws enacted or bills content yield no difference when it comes to measuring pro-change SIG influence.

**Remark 1.** In a separating equilibrium, \(b^*(\zeta^A, \zeta^P(s)) = 1 > b^*(\zeta^A, \zeta^P(w)) = b_w\) and \(E(y^*(\zeta^A, \zeta^P(s))) = 1 - p > y^*(\zeta^A, \zeta^P(w)) = b_w\).

The next proposition characterizes the pro-change SIG’s strategy in a separating equilibrium: \(\zeta^P(\tau), \ i^P(\tau), \ \tau \in \{s, w\}\) assuming without loss of generality that the SIG announces its type \((m(\tau) = \tau)\). To this end, it is useful to define the following quantity:

\[
\bar{l}_i^P(c^P_s, b_w) := \frac{c^P_s - (1 - p - b_w)}{c^P_i - c^P_s}.
\]

**Proposition 1.** In a separating equilibrium, the pro-change SIG’s equilibrium strategy satisfies:

1. \(\zeta^P(s) = (s, 0)\) and \(\zeta^P(w) = (w, l^P_i(w)), \) with \(l^P_i(w) = \max \left\{0, \bar{l}_i^P(c^P_s, b_w)\right\};\)

2. \(l^P_i(s) = 1\) and \(l^P_i(w) = 0\).

In a separating equilibrium, a strong pro-change SIG incurs no inside lobbying expenditures: \(l^P_i(s) = 0\). To understand this result, notice that the decision-maker wants the SIG to subsidize the cost of defending her proposal, whereas the pro-change SIG wants the decision-maker to pay this cost in full. A strong pro-change SIG has no incentive to pay a cost at the inside lobbying stage to reveal it is willing to engage in costly outside lobbying. Consequently, only a weak pro-change SIG sometimes incurs inside lobbying expenditures to credibly signal it is not able to bear the cost of outside lobbying activities. Inside lobbying expenditures serve to credibly ‘plead poverty’. Costly signaling, however, is not always necessary. When a strong pro-change SIG’s lobbying cost is low \((c^P_s \leq 1 - p - b_w)\), it strictly prefers the risky bill \(b = 1\) to the compromise bill \(b = b_w\). Hence, a cheap talk message by the strong pro-change SIG is credible and a separating equilibrium exists absent any inside lobbying expenditure.\(^{14}\)

\(^{14}\)Observe that imposing the Intuitive Criterion guarantees that the separating equilibrium with the minimum level of expenditures is selected.
Proposition 1 considers separately inside and outside lobbying expenditures and thus cannot characterize the relationship between money and influence. The next remark shows that a strong pro-change SIG’s total observed expenditures \( l^P_p(s) + l^o_p(s) \) are always higher than a weak one’s.

**Remark 2.** In a separating equilibrium, the following inequality always holds:
\[
l^P_p(s) + l^o_p(s) > l^P_p(w) + l^o_p(w)
\]

As the separating equilibrium exists only under specific conditions, it is essential to characterize conditions for existence of a pooling equilibrium. In a pooling equilibrium, the decision-maker always chooses \( b = b_w \). As out-of-equilibrium beliefs are restricted by the Intuitive Criterion, a pooling equilibrium exists whenever there does not exist a deviating signaling strategy satisfying (i) the voter anticipates only a strong type deviates and, given the voter’s belief, (ii) only a strong pro-change SIG prefers to deviate. In a separating equilibrium, only a weak pro-change SIG incurs inside lobbying expenditures. Therefore, a profitable signaling deviation can only occur when cheap talk messages credibly reveal an SIG’s strength. By Lemma 1 and Proposition 1, a pooling equilibrium therefore exists whenever \( \min\{c^P_p, k/2\} \geq 1 - p - b_w \).

For given parameter values, there might exist (infinitely) many pooling equilibria. These equilibria differ only in the level of inside lobbying expenditures determined by the decision-maker’s out-of-equilibrium belief. To see this, suppose that the decision-maker believes that the pro-change SIG is strong when she observes no inside lobbying expenditures (an out-of-equilibrium event). Whenever \( k/2 \leq 1 - p - b_w \) (so the decision-maker does not always prefer \( b_w \)), she would choose \( b = 1 \) and asks for help following \( l^P_i = 0 \). As this would impose a high cost on both strong and weak types, the pro-change SIG is willing to incur inside lobbying expenditures so that the decision-maker compromises with the anti-change SIG.\(^{15}\) In contrast, a pro-change SIG never engages in outside lobbying in a pooling equilibrium since the decision-maker always compromises.

**Proposition 2.** A pooling equilibrium exists if and only if \( \max\{c^P_p, k/2\} \geq 1 - p - b_w \). The decision-maker always chooses \( b^* = b_w \). The pro-change SIG’s equilibrium strategy satisfies for \( \tau \in \{s, w\} \):

1. \( \zeta^P\tau = (m(\tau), l^P\tau) \), with \( m(\tau) \in \{s, w\} \) and \( l^P\tau = 0 \) if \( k/2 > 1 - p - b_w \) and \( l^P\tau = \left[0, \min\left\{l^P_i(c^P_p, b_w), b(w)c^P_w\right\}\right] \) otherwise;

\(^{15}\)Importantly, while the Intuitive Criterion restricts signaling (inside lobbying) expenditures in a separating equilibrium (see footnote 14), it imposes no restriction on signaling expenditures in a pooling equilibrium.
2. \( l^*_P(\tau) = 0 \).

Table 1 summarizes this section’s main theoretical findings by distinguishing between four different cases (omitting boundary cases and ignoring arguments for ease of exposition). The first case corresponds to \( k/2 > 1 - p - b_w \) when only a pooling equilibrium with no lobbying expenditure exists. The second case corresponds to \( k/2 < 1 - p - b_w \) and \( c^P_s > \bar{c} \) when only a pooling equilibrium with (possibly) positive inside lobbying expenditures exist. The third case corresponds to \( k/2 < 1 - p - b_w < c^P_s \leq \bar{c} \) when the separating equilibrium and pooling equilibria can arise. All equilibria can exhibit positive level of inside lobbying expenditures. The last case corresponds to \( \max\{c^P_s, k/2\} < 1 - p - b_w \) when only a separating equilibrium with no inside lobbying expenditure exists.

<table>
<thead>
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<th>Case</th>
<th>Type</th>
<th>Sep.</th>
<th>Pool.</th>
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<tbody>
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(a) Inside lobbying

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(b) Outside lobbying

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(c) Policy choice

Table 1: Equilibrium strategies

‘Sep.’ stands for separating equilibrium; ‘Pool.’ for pooling equilibria. The different cases are described in the text. In Tables 1a and 1b for each case, the first line s (second line w) describes a strong (weak) pro-change SIG’s strategy. In Table 1c, for each case, the first (second) line corresponds to the decision-maker’s bill choice after observing a strong (weak) pro-change SIG’s signal: \( \zeta^P(s) \) (\( \zeta^P(w) \)).

To understand the empirical implications of these results, consider an empirical researcher interested in assessing a pro-change SIG’s ability to tilt policy choice \( b \) in its favor. For simplicity, assume that the researcher’s data set includes the full population of pro-change SIGs. A researcher using exclusively inside lobbying expenditures as a proxy for influence would then run the following

\[\text{In particular, assume that all the cases described in Table 1 are included in the data set and for each } c^P_s, c^P_w, b_w, k, p, \text{ there are } N > 1 \text{ SIGs, } q'' \text{ of them being strong and } N' < N \text{ playing a separating strategy profile when separating and pooling equilibria can arise.}\]
regression (with \( b \) the content of a bill, \( X^P \) a set of controls, and \( \epsilon^P \) the residual):

\[
b = \beta_0^P + \beta_1^P l_1^P + \beta_2^P X^P + \epsilon^P
\]  

(4)

This section’s theoretical results suggest that the resulting \( \hat{\beta}_1^P \) is a biased estimate of SIG influence even under the (arguably unrealistic) assumption that an empirical researcher can distinguish between the four cases detailed in Table 1. Notice first that the researcher would underestimate the extent of the pro-change SIG influence. Regression (4) yields a coefficient only in case 3. But while a pro-change SIG has no influence on policy choice in cases 1 and 2, it biases political decisions in case 4 through cheap talk message. Furthermore, the sign \( \hat{\beta}_1^P \) is likely to be negative and the researcher may mistakenly conclude that a pro-change SIG is better off when it does not intervene at the inside lobbying stage since in case 2, \( \hat{\beta}_1^P \) is negative.

Importantly, the empirical researcher cannot just correct for the sign as (s)he is likely to underestimate the pro-change SIG influence. Fixing parameter values, multiple pooling equilibria exist differing only in the amount of inside lobbying expenditures. Since these expenditures have no effect on policy choice, the estimate \( \hat{\beta}_1^P \) suffers from attenuation bias. The risk of attenuation bias does not require that inside lobbying expenditures be positive in (some) pooling equilibria. It is also present when the researcher can control for the pro-change SIG’s type. The attenuation bias is eliminated only if the researcher can control for equilibrium selection.

Importantly, the researcher can leverage the pro-change SIG’s equilibrium behavior to determine which equilibrium is being played. As Table 1 describes, a strong pro-change SIG engages in outside lobbying only if it plays a separating strategy and thus tilts policy choice in its favor. That is, in the present theoretical framework, the correct specification to measure SIG influence is:

\[
b = \alpha_0^P + \alpha_1^P l_1^P + \alpha_2^P X + \nu^P
\]  

(5)

This section therefore highlights that to obtain an unbiased estimate of pro-change SIG influence, the distribution of parameter values and the proportion of observations associated with each case and equilibrium would determine whether the researcher recovers a statistically significant estimate at conventional levels.

To see that observing types is not sufficient, consider the following regression: \( b = \gamma_0^P + \gamma_1^P l_{\{\tau=s\}} + \gamma_2^P X^P + \eta^P \) with \( l_{\{\tau=s\}} \) an indicator function equals to 1 when \( \tau = s \). The researcher would correctly obtain \( \gamma_1^P > 0 \) in cases 1 and 2. However, in case 2, the estimate would suffer from attenuation bias since an SIG’s type does not provide information about the equilibrium being played.
empirical researchers need to consider SIGs’ outside lobbying activities rather than (as it is common) their inside lobbying expenditures.

**Anti-change SIG influence**

In this section, I focus on the anti-change SIG. To this end, I assume that \( q^P = 0 \), so the pro-change SIG’s signal (\( \zeta^P \)) has no impact on the decision-maker’s policy choice and the decision-maker never asks for help under Assumption 2. Further, I now assume \( q^A > 0 \) so the decision-maker is uncertain ex-ante about the anti-change SIG’s strength.

The decision-maker now chooses between a radical change \( b = 1 \), a compromise bill \( b_w \) which makes a weak type indifferent between engaging in outside lobbying or not, or a compromise bill \( b_s := \frac{\zeta^A}{p} \), which renders a strong type indifferent between \( l^A_o = 1 \) and \( l^A_o = 0 \). The anti-change SIG biases policy choice whenever the decision-maker prefers the minimal change \( b_s \). Influence can take two distinct forms. First, the threat of outside lobbying may induce the decision-maker to choose the compromise bill \( b = b_s \). Second, a strong anti-change SIG credibly reveals its strength at the inside lobbying stage to induce the decision-maker to propose \( b = b_s \) (a weak type, in turn, obtaining \( b = b_w \)). Observe that inside lobbying expenditures are correlated with influence only in the second case; that is, when a separating equilibrium exists (\( \zeta^A(s) \neq \zeta^A(w) \)). Lemma 2 characterizes necessary and sufficient conditions for a separating equilibrium to exist.

**Lemma 2.** A *separating equilibrium* exists if and only if:

\[
\max \left\{ 1 - p - k, \frac{k}{1 - p} \right\} \leq b_s \leq (1 - p) b_w
\]

A separating equilibrium exists only if a strong anti-change SIG’s benefit from differentiation is greater than the associated cost. The benefit from differentiation is positive only if the decision-maker chooses \( b_s \) after learning the anti-change SIG is strong. Compromising with a strong type, however, is the decision-maker’s best response only if \( b_s \geq \max \left\{ 1 - p - k, \frac{k}{1 - p} \right\} \). When \( b_s < 1 - p - k \), the decision-maker prefers to propose \( b = 1 \) and face the costly lottery induced by the anti-change SIG’s outside lobbying activities than to compromise. When \( b_s < \frac{k}{1 - p} \Leftrightarrow (1 - p) b_s - k < 0, \)

\[19\] It can be checked that any other proposal can only reduce the decision-maker’s policy payoff if the bill is enacted without diminishing the likelihood of outside lobbying activities.
the decision-maker is not credible when she proposes $b = b_s$ since she is not willing to defend her proposal if the SIG engages in outside lobbying. By choosing $l_o^A = 1$, the SIG then gets a payoff of $-c_s^A$ as the bill is abandoned with certainty ($y = 0$). By choosing $l_o^A = 0$, it gets $-b_s = -c_s^A/p$ since the compromise bill is passed for sure. Obviously, a strong anti-change SIG strictly prefers to engage in outside lobbying and the decision-maker anticipating this proposes her preferred bill ($b = 1$).

A second necessary condition for the existence of a separating equilibrium is that a strong anti-change SIG is willing to reveal its type. Even though the single-crossing condition holds ($c_s^A < c_w^A$), this is not always guaranteed. When a strong anti-change SIG reveals its strength, the policy-maker proposes $b = b_s$. When it pretends to be weak, the decision-maker chooses $b = b_w$, but the anti-change SIG then engages in outside lobbying and the probability that the bill is enacted is only $1 - p$.

The expected payoff from imitating a weak type is thus: $-(1 - p)b_w - c_p^P$. Due to the anti-change SIG’s outside option of engaging in outside lobbying, the full benefit from differentiation is strictly lower than the difference in policy choices. It is equal to: $-b_s - (1 - p)b_w - c_p^P = (1 - p)(b_w - b_s)$.

When a weak anti-change SIG imitates a strong type, in contrast, it obtains a bill closer to the status quo and enjoys the full benefit of the difference in policy choices. Its benefit from imitating a strong type is thus: $b_w - b_s$. Consequently, a weak type’s benefit from imitation is always strictly greater than a strong type’s benefit from differentiation. This implies that absent a lower lobbying cost for the strong type, a separating equilibrium does not exist. By continuity, the same holds true when $c_s^A$ is relatively close to $c_w^A$ as a strong SIG’s lower cost of lobbying is not enough to compensate for the smaller benefit from differentiation. A separating equilibrium exists only if lobbying costs satisfy $c_s^A \leq (1 - p)c_w^A$.

The next proposition characterizes the anti-change SIG’s strategy in a separating equilibrium: $\zeta_s^A(\tau), \; l_o^A(\tau), \; \tau \in \{s, w\}$ assuming without loss of generality that the anti-change SIG announces its type ($m(\tau) = \tau$). Only a strong anti-SIG incurs strictly positive inside lobbying expenditures since it always obtains the more favorable bill $b_s$. Since the decision-maker always compromises with the SIG on the equilibrium path, no type engages in outside lobbying. Denoting $l_i^{A\text{sep}}(b_s) := \frac{b_w - b_s}{c_w^P}$, I obtain:

**Proposition 3.** In a separating equilibrium, the anti-change SIG’s equilibrium strategy satisfies:

1. $\zeta_s^A(s) = (s, l_i^A(s))$ and $\zeta_s^A(w) = (w, l_i^A(w))$, with $l_i^A(s) = l_i^{A\text{sep}}(b_s)$ and $l_i^P(w) = 0$;
2. \( l_o^A(\tau) = 0, \ \tau \in \{s, w\} \).

Proposition 3 indicates that in a separating equilibrium, inside lobbying expenditures are perfectly positively correlated with influence. However, as established in Lemma 2, a separating equilibrium does not always exist. Therefore, it is critical to study the extent of the anti-change SIG influence absent informative inside lobbying expenditures.

By the Intuitive Criterion, as explained above, a pooling equilibrium does not exist when only a strong type has a profitable signaling deviation. This is the case when the conditions described in Lemma 2 hold and absent any information at the inside lobbying stage, the decision-maker prefers \( b_w \) to \( b_s \). A strong anti-change SIG would then be better off by incurring inside lobbying expenditures \( l_i^{sep}(s) \), which would credibly reveals its type and induces the decision-maker to propose \( b_s \). In all other cases, however, the Intuitive Criterion does not rule out the existence of pooling equilibria.

In a pooling equilibrium, the decision-maker’s choice between \( b_s, b_w \), and 1 depends on two factors: (i) her assessment of the threat of outside lobbying activity as measured by her prior \( q^A \) that the SIG is strong and (ii) the strong anti-change SIG’s lobbying cost. When a strong anti-change SIG’s fund-raising cost is low (\( b_s < \max\{1 - p - k, k/(1 - p)\} \)), the decision-maker never wants to or cannot credibly compromise with a strong SIG. Therefore she always chooses between \( b = b_w \) and \( b = 1 \). The decision-maker gains little from compromising with the weak anti-change SIG when the probability the SIG is strong (\( q^A \)) is high since she faces a high risk of outside lobbying activities. Hence, she proposes \( b_w \) only if \( q^A \) is sufficiently low. When a strong anti-change SIG’s lobbying cost is relatively high (\( b_s \geq \max\{1 - p - k, k/(1 - p)\} \)), \( b_s \) always dominates \( b = 1 \). The decision-maker benefits from compromising with a strong anti-change SIG is high whenever the risk of outside lobbying is high. Hence, as before, she proposes \( b_w \) only if \( q^A \) is low. These results are summarized in Lemma 3 in which I impose \( (1 - p)b_w > \max\{1 - p - k, \frac{k}{1 - p}\} \) to limit the number of cases (an amended statement holds when the inequality is reversed).

**Lemma 3.** Suppose \( (1 - p)b_w > \max\{1 - p - k, \frac{k}{1 - p}\} \). There exists a unique \( q^A: (0, 1) \rightarrow (0, 1) \) such that a pooling equilibrium exists if and only if \( b_s \notin \left[ \max\{1 - p - k, \frac{k}{1 - p}\}, (1 - p)b_w \right] \) or \( q^A > q^A(b_s) \). Furthermore, there exists a unique \( q^A \in (0, 1) \) such that in a pooling equilibrium, the decision-maker’s policy choice satisfies:

\[^{20}\text{Notice that it is impossible to uniquely characterize the decision-maker’s equilibrium strategy when } b_s = \max\{1 - p - k, \frac{k}{1 - p}\}.\]

20
1. When $b_s \leq \max \left\{ 1 - p - k, \frac{k}{1-p} \right\}$, $b^* = b_w$ if and only if $q^A \leq q^A$ and $b^* = 1$, otherwise;
2. When $b_s \geq \max \left\{ 1 - p - k, \frac{k}{1-p} \right\}$, $b^* = b_w$ if and only if $q^A \leq q^A(b_s)$ and $b^* = b_s$, otherwise.

The next proposition characterizes the anti-change SIG’s equilibrium strategy in a pooling equilibrium. First, as the decision-maker does not always compromise, the anti-change SIG sometimes engages in outside lobbying on the equilibrium path. Furthermore, the anti-change SIG may incur inside lobbying expenditures even though they have no impact on the decision-maker’s policy choice. This result is again driven by the decision-maker’s out-of-equilibrium belief. Absent inside lobbying expenditures (an out-of-equilibrium event), the decision-maker would choose a more extreme policy than when she observes inside lobbying expenditures. The anti-change SIG then incurs inside lobbying expenditures to induce a compromise. To state the next proposition, it is useful to define $\bar{t}_i^{A^o} := \frac{(1-p)(1-b_w)}{c^d}$.

**Proposition 4.** Suppose $(1-p)b_w > \max \left\{ 1 - p - k, \frac{k}{1-p} \right\}$. In a pooling equilibrium, the anti-change SIG’s equilibrium strategy satisfies $m(s) = m(w) \in \{s, w\}$ and:

1. When $b_s \leq \max \left\{ 1 - p - k, \frac{k}{1-p} \right\}$,
   
   i. If $q^A \leq q^A$, $l_i^{A^o}(s) = l_i^{A^o}(w) \in \left[ 0, \bar{t}_i^{A^o} \right]$ and $l_o^{A^o}(s) = 1$, $l_o^{A^o}(w) = 0$;
   
   ii. If $q^A > q^A$, $l_i^{A^o}(s) = l_i^{A^o}(w) = 0$ and $l_o^{A^o}(s) = l_o^{A^o}(w) = 1$;

2. When $b_s \geq \max \left\{ 1 - p - k, \frac{k}{1-p} \right\}$,
   
   i. If $q^A \leq q^A(b_s)$, $l_i^{A^o}(s) = l_i^{A^o}(w) = 0$ and $l_o^{A^o}(s) = 1$, $l_o^{A^o}(w) = 0$;
   
   ii. If $q^A > q^A(b_s)$, $l_i^{A^o}(s) = l_i^{A^o}(w) \in \left[ 0, \bar{t}_i^{A^o}(b_s) \right]$ and $l_o^{A^o}(s) = l_o^{A^o}(w) = 0$.

Table 2 summarizes this section’s main findings under the assumption that $(1-p)b_w > \max \{1-p-k, \frac{k}{1-p}\}$ (omitting arguments and boundary cases for ease of exposition). Cases 1 and 2 correspond to $b_s < \max \left\{ 1 - p - k, \frac{k}{1-p} \right\}$ when only a pooling equilibrium exists and are differentiated by the value of the decision-maker’s posterior ($q^A < q^A$ and $q^A > q^A$, respectively). Cases 5 and 6 correspond to $b_s > (1-p)b_w$ when again only a pooling equilibrium exists and are differentiated by the value of the decision-maker’s prior ($q^A < q^A(b_s)$ and $q^A > q^A(b_s)$, respectively). Finally, cases 3 and 4 correspond to $b_s \in \left( \max \left\{ 1 - p - k, \frac{k}{1-p} \right\}, (1-p)b_w \right)$ when a separating equilibrium exists. In case 3, the decision-maker’s prior satisfies $q^A < q^A(b_s)$ so a pooling equilibrium does not exist.
### Table 2: Equilibrium strategies

‘Sep.’ stands for separating equilibrium; ‘Pool.’ to pooling equilibrium. The different cases are described in the text.

In Tables 2a and 2b, for each case, the first line $s$ (second line $w$) describes a strong (weak) pro-change SIG’s strategy.

In Table 2c, for each case, the first (second) line corresponds to the decision-maker’s bill choice after observing a strong (weak) anti-change SIG’s signal: $\zeta_A(s)$ ($\zeta_A(w)$).

To study the empirical implications of this section’s results, consider an empirical researcher who seeks to measure anti-change SIGs’ influence on policy choice $b$. For simplicity, I assume again that the empirical researcher’s data set contains the full population of SIGs with multiple observations for each parameter value and a vector of independent variables $X^d$ to control for the anti-change SIGs’ (potential) lobbying costs. To show the issues associated with using inside lobbying expenditures as
a measure of influence, assume the researcher runs the following regression (with $\epsilon^A$ the residual):

$$b = \beta_0^A + \beta_1^A l_i^A + \beta_2^A X^A + \epsilon^A$$

(6)

Regression (6) would correctly identify that inside lobbying expenditures are positively correlated with influence. That is, it would yield $\hat{\beta}_1^A \leq 0$. However, the estimate obtained would suffer from attenuation bias even if the empirical researcher can control for the decision-maker’s prior and therefore can fully distinguish between the six cases described in Table 2. This issue again arises because the researcher faces an equilibrium selection problem: (s)he cannot distinguish whether the anti-change SIG plays a separating or pooling strategy in case 4. Notice that in this case, the anti-change SIG obtains its preferred policy (even if weak) through the threat of outside lobbying activities. As such, the downward bias of $\hat{\beta}_1^A$ might be severe. As such, the present theoretical framework suggests that estimates regarding the impact of lobbying on effective tax rate (Richter et al., 2009) or contributions on trade protection (Maggi and Goldberg, 1999), where SIGs arguably oppose changes, should be understood as lower bounds on SIG influence.

The inclusion of outside lobbying expenditures does not help correct for the identified attenuation bias. This does not imply that outside lobbying activities are of no use for the empirical researcher. They play a critical role when the (arguably unrealistic) assumption that the researcher can control for $q^A$ is relaxed; that is, the researcher cannot distinguish between cases 1 and 2, between cases 3 and 4, and between cases 5 and 6. Outside lobbying expenditures can then be used as a proxy for the decision-maker’s prior. As Table 2a illustrates, outside lobbying activities are more likely when the decision-maker does not compromise (i.e., choose the most extreme proposal in her choice set in a pooling equilibrium). Incorporating outside lobbying expenditures in the regression therefore allows the researcher to identify situations when the anti-change SIG does not influence policy choices. But this directly implies that the empirical researcher can only recover the extent of the anti-change SIG influence with outside lobbying expenditures. These expenditures provide no information regarding how these SIGs tilt policy choices in their favor (threat of outside lobbying or inside lobbying activities). This paper thus suggests there exist limits to our ability to understand anti-change SIG influence due to their strategic use of both inside and outside lobbying.
Policy choices with competing SIGs

In this section, I discuss the case when both SIGs' strength is unknown to the decision-maker (i.e., $q^A > 0$ and $q^P > 0$) and relegate the formal analysis of this set-up to Supplemental Appendix C. Both SIGs now have an opportunity to affect the decision-maker’s policy choice: there is competition for influence. This competition, however, does not change the key insights from the previous sections.

When a pro-change SIG plays a separating strategy, a weak type uses inside lobbying expenditures to plead poverty credibly. There also exist pooling equilibria with positive level of inside lobbying expenditures. Further, a separating and pooling equilibria can arise for some parameter values. Inside lobbying expenditures, therefore, are still a poor proxy for pro-change SIG influence. Outside lobbying expenditures, on the other hand, remain an unbiased measure of influence.

An anti-change SIG plays a separating strategy on the equilibrium path if and only if the compromise bill $b_s$ takes intermediary values. The decision-maker, however, has less incentive to compromise since the pro-change SIG may be able to help. The decision-maker therefore sometimes chooses the radical bill $b = 1$ even after learning the anti-change SIG is strong. As a result, the set of parameter values such that the anti-change SIG plays a separating strategy profile is lower (in the sense of set inclusion) than absent competition for influence since the expected benefit from differentiation is lower. Furthermore, even when it plays a separating strategy, the anti-change SIG may engage in outside lobbying. This means that inside lobbying expenditures are no longer always positively correlated with influence and regression might yield wrongly signed estimate. Outside lobbying expenditures, in contrast, still provide an unbiased estimate of the extent of anti-change SIG influence even when these SIGs play a separating strategy.

Conclusion

In this paper, I consider a model in which SIGs can use both inside lobbying—to shape the content of a bill—and outside lobbying—to affect the fate of a bill. I show that inside lobbying expenditures are a poor proxy for SIG influence. Empirical analyses focusing exclusively on this type of expenditures are likely to produce downwardly biased estimates and possibly wrongly signed estimates. The
small impact of money on political decisions generally documented is thus not inconsistent with the popular wisdom that SIGs bias the political process in their favor. To evaluate the empirical validity of this popular wisdom, one needs to consider the right type of SIG expenditures as only outside lobbying expenditures may permit to correctly identify the extent of SIG influence.

Several interesting extensions would help refine our understanding of SIGs’ ability to bias policy choices in their favor. The present manuscript supposes that outside lobbying activities only affect the likelihood a bill is enacted into law. However, outside lobbying can also change the salience of an issue and thus the decision-maker’s policy agenda. Future theoretical work would do well to consider the strategic interactions between pre- and post-policy choice outside lobbying. This paper also abstains from studying the economic consequences of SIG influence. Analyzing the impact of outside lobbying activities in important policy domains such as income redistribution, trade protection, or environmental regulation constitutes promising avenues for future research.
Appendix: Proofs

I first introduce some notation. Recall $\zeta^l(\tau) \in \{s, w\} \times \mathbb{R}_+$ is an SIG’s signal as a function of her type $\tau \in \{s, w\}$; $J \in \{A, P\}$. Throughout, I assume without loss of generality that when an SIG plays a separating strategy, it announces its type: $m(\tau) = \tau$, $\tau \in \{s, w\}$. The decision-maker’s posterior that a pro-change (resp., anti-change) SIG is strong following its signal is $\mu^P(\zeta^P)$ (resp., $\mu^A(\zeta^A)$). As I restrict attention to pure strategy in the main text, posterior always satisfies $\mu^J(\zeta^J) \in \{0, q^J, 1\}$, $J \in \{A, P\}$. Denote $b(\zeta^P, \zeta^A) \in [0, 1]$ and $d(\zeta^P, \zeta^A, b, l^A_o) \in \{0, 1, 2\}$ the decision-maker’s strategy (resp. policy choice and defense of her proposal) as a function of SIGs’ signals, her policy choice, and the anti-change SIG’s outside lobbying activity. Denote $l^A_o(b, \zeta^P; \tau) \in \{0, 1\}$ the anti-change SIG’s outside lobbying strategy as a function of the decision-maker’s proposal, pro-change SIG’s signal, and its own type. Similarly, denote $l^A_o(b, l^A, d; \tau) \in \{0, 1\}$ the pro-change SIG’s outside lobbying strategy as a function of the decision-maker’s proposal, anti-change SIG’s outside lobbying activities, decision-maker’s defense strategy, and its own type. Starred strategies denote equilibrium strategies.

In the proofs, I focus on the SIGs’ inside lobbying strategy with players playing their best response down the game tree. This implies in particular (assuming $(1 - p)b - k \geq 0$ so $d^*(\cdot, l^A_o = 1) \neq 0$): i) decision-maker’s defense strategy satisfies: $d^*(\zeta^P, \zeta^A, 1) = 1$ if and only if $\mu^P(\zeta^P) = 1$ and $(1 - p)b - c^P_s \geq 0$ (so a strong pro-change SIG has incentive to engage in outside lobbying), and $d^*(\zeta^P, \zeta^A, 1) = 2$ otherwise; ii) the anti-change SIG chooses $l^A_o(b, \zeta^P; \tau) = 1$ if and only if $-(1 - p)b - c^A_t > -b$, $\forall b \in [0, 1]$. Notice as well that the decision-maker’s equilibrium proposal satisfies $b^*(\cdot) \in \{b_s, b_w, 1\}$, with $b_s = \frac{c^A_t}{p}$, $\tau \in \{s, w\}$. Any other choice decreases her payoff conditional on the bill being enacted and weakly reduces the likelihood the proposal is passed.

I now prove the results regarding the pro-change SIG influence (with $\zeta^A$ the anti-change SIG’s uninformative signal). The decision-maker’s equilibrium strategy satisfies $b^*(\cdot) \in \{b_w, 1\}$. It can be checked that by Assumptions 1 and 2 when $\zeta^P(s) = \zeta^P(w) = \zeta^P$ (so $\mu^P(\zeta^P) = q^P$), then the decision-maker’s equilibrium strategy satisfies: $b^*(\zeta^P, \zeta^A) = b_w$ and $d^*(\zeta^P, \zeta^A, b_w, 0) = 0$, $d(\zeta^P, \zeta^A, b_w, 1) = 2$. In turn, the anti-change SIG’s strategy is $l^A_o(b_w, \zeta^P; \tau) = 0$ for all $\tau \in \{s, w\}$.

**Lemma 4.** The pro-change SIG plays a separating strategy on the equilibrium path (i.e. $\zeta^P(s) \neq \zeta^P(w)$) only if: $b(\zeta^P(s), \zeta^A) = 1$ and $b(\zeta^P(w), \zeta^A) = b_w$. 

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Proof. The proof is by contradiction. Suppose \( b^{\ast} (\zeta^P(s), \zeta^A) = 1 = b^{\ast} (\zeta^P(w), \zeta^A) \) so \( l_o^{A\ast}(1, \zeta^P; \tau) = 1 \) for all \( \zeta^P, \tau \). The decision-maker’s best response is then \( d^\ast (\zeta^P(s), \zeta^A, 1, 1) = 1 \) and \( d^\ast (\zeta^P(w), \zeta^A, 1, 1) = 2 \). The strong and weak types’ incentive compatibility constraints are then, respectively: \( (1 - p) - c_s^P - c_s^P l_i^P(s) \geq (1 - p) - c_s^P l_i^P(w) \) and \( (1 - p) - c_w^P l_i^P(w) \geq 0 - c_w^P l_i^P(s) \). A necessary condition for existence of such equilibrium is \( 1 \leq l_i^P(w) - l_i^P(s) \leq \frac{(1 - p)(1 - c_s^P)}{c_w^P} \), which is never satisfied by Assumption 2. A similar reasoning directly implies that \( b(\zeta^P(s), \zeta^A) = b_w \) and \( b(\zeta^P(w), \zeta^A) = 1 \) cannot be an equilibrium strategy. Finally, when \( b(\zeta^P(s), \zeta^A) = b_w = b(\zeta^P(w), \zeta^A) \), the pro-change SIG’s signal has no effect on the decision-maker’s strategy on the equilibrium path (since \( l_o^{A\ast}(b_w, \zeta^P; \tau) = 0 \) and \( d^\ast (\zeta^P, \zeta^A, b_w, 0) = 0 \) for \( \zeta^P \in \{\zeta^P(s), \zeta^P(w)\} \)). The equilibrium restriction then imposes the pro-change SIG plays a pooling strategy.

A direct consequence of Lemma 4 is that whenever the pro-change SIG plays a separating strategy, a strong type’s equilibrium outside lobbying strategy satisfies \( l_o^{P\ast}(b, l_o^{A\ast}, d^\ast; s) = 1 \) since \( b^{\ast} (\zeta^P(s), \zeta^A) = 1 \) and \( \mu^P (\zeta^P(s)) = 1 \) so \( l_o^{A\ast} (\cdot) = 1 \) and \( d^\ast (\zeta^P(s), \zeta^A, 1, 1) = 1 \).

Lemma 5. The pro-change SIG plays a separating strategy on the equilibrium path only if: \( l_i^P(s) = 0 \).

Proof. The proof is by contradiction. Suppose \( \zeta^P(s) = (s, l_i^P(s)) \) with \( l_i^P(s) > 0 \). By the Intuitive Criterion, this implies \( l_i^P(w) = 0 \). By Lemma 4 a weak pro-change SIG’s expected policy payoff is \( b_w \) when she sends signal \( \zeta^P(w) \) and 0 when she sends signal \( \zeta^P(s) \). Consequently, a weak pro-change SIG has no incentive to send a signal \( \zeta^P \) satisfying \( l_i^P > 0 \). By the Intuitive Criterion, the strong type has a profitable deviation to \( \hat{l_i^P} \in (0, l_i^P(s)) \), a contradiction.

Proof of Lemma 4. Necessity. Suppose \( \zeta^P(s) \neq \zeta^P(w) \). When \( b_w > 1 - p - k/2 \), the decision-maker’s best response is: \( b(\zeta^P, \zeta^A) = b_w \) for all \( \zeta^P \). By Lemma 4 a separating equilibrium does not exist. Suppose \( b_w \leq 1 - p - k/2 \) so the decision-maker’s best response satisfies \( b(\zeta^P(s), \zeta^A) = 1 \) and \( b(\zeta^P(w), \zeta^A) = b_w \). Using Lemma 5 a strong pro-change SIG’s incentive compatibility constraint (IC) is: \( (1 - p) - c_s^P \geq b_w - c_s^P l_i^P(w) \). The weak type’s (IC) is: \( b_w - c_w^P l_i^P(w) \geq 0 \). By the Intuitive Criterion, \( l_i^P(w) = \max \left\{ \frac{c_s^P - b_w}{c_s^P}, 0 \right\} \). Therefore, both (IC) are automatically satisfied whenever \( c_s^P \leq (1 - p) - b_w \) (so \( l_i^P(w) = 0 \)). When \( c_s^P > (1 - p) - b_w \), a weak type’s (IC) is satisfied if and only if \( c_s^P \leq c^P(b_w, c^P_{w}) \), with \( c^P(b_w, c^P_{w}) := \frac{c_w^P (1 - p) - b_w}{c_w^P - b_w} \), with \( c^P(b_w, c^P_{w}) < (1 - p) \) as claimed.
Sufficiency. Suppose \( b_w \leq 1 - p - k/2 \) and \( c_s^P \leq \bar{c}_P(b_w, c_w^P) \), and consider the following assessment:
i) A weak (strong) pro-change SIG’s signal is \( \zeta^P(w) = (w, l^P_i(w)) \) \((\zeta^P(s) = (s, 0))\), with \( l^P_i(w) = \max \{\frac{c_s^P + b_w - (1-p)}{c_s^P}, 0\} \); ii) The decision-maker’s posterior is: \( \mu^P(\zeta^P) = 0 \) if \( \zeta^P = (w, l^P_i) \), with \( l^P_i \geq l^P_i(w) \), and 1 otherwise; iii) the decision-maker’s policy choice is: \( b(\zeta^P, \zeta^A) = b_w \) if \( \zeta^P = (w, l^P_i) \), with \( l^P_i \geq l^P_i(w) \) and \( b(\zeta^P, \zeta^A) = 1 \), otherwise; (iv) all players play their best response down the game tree (see above). It can be checked that beliefs satisfy Bayes’ rule, the decision-maker’s policy choice is a best response given her belief, and the pro-change SIG’s (IC) hold. Hence, the assessment described above is an equilibrium.

Proof of Proposition 7. Direct from the proof of Lemma 1.

The proofs of Remarks 1 follows directly from the proof of Lemma 1. The proof of Remark 2 follows from noticing that outside lobbying expenditures are normalized to 1 and by Lemma 1 \( l^P_i(w) < 1 \).

Proof of Proposition 2. The proof of existence follows directly from Lemma 3 and noticing that a weak type has no incentive to imitate a strong type when \( b(\zeta^P(s), \zeta^A) = 1 \) and \( d(\zeta^P(s), \zeta^A, b, 1) = 1 \). Therefore, when \( c_s^P < 1 - p - b_w \) and \( b_w < 1 - p - k/2 \), only a strong type has incentive to send signal \( \hat{\zeta}^P(s) = (s, 0) \) when \( \mu^P(\hat{\zeta}^P(s)) = 1 \). By the Intuitive Criterion, a pooling equilibrium does not exist then. It exists in all other cases.

Suppose \( 1-p-b_w < k/2 \). The decision-maker’s best responses are \( b^*(\zeta^P, \zeta^A) = b_w \) and \( d^*(\zeta^P, \zeta^A, b_w, 0) = 0 \), \( d(\zeta^P, \zeta^A, b_w, 1) = 2 \) for all \( \zeta^P \). The only equilibrium signal is then \( \zeta^P(s) = \zeta^P(w) = (m, 0) \) for some \( m \in \{s, w\} \). Suppose \( k/2 \leq 1 - p - b_w \) and consider the following belief structure for the decision-maker: \( \mu^P(\zeta^P) = 1 \), when \( \zeta^P = (m, l^P_i) \) for \( m \in \{s, w\} \) and \( l^P_i \in [0, \hat{l}^P_i) \), with \( \hat{l}^P_i > 0 \), and \( \mu^P(\zeta^P) = q^P \), otherwise. Given this belief structure, the decision-maker’s best response is: \( (b^*(\zeta^P, \zeta^A) = 1, d^*(\zeta^P, \zeta^A, 1, 0) = 0, d^*(\zeta^P, \zeta^A, 1, 1) = 1) \), \( \forall \zeta^P \in \{s, w\} \times [0, \hat{l}^P_i) \) and \( (b^*(\zeta^P, \zeta^A) = b_w, d^*(\zeta^P, \zeta^A, b_w, 0) = 0, d^*(\zeta^P, \zeta^A, b_w, 1) = 2) \), \( \forall \zeta^P \in \{s, w\} \times [\hat{l}^P_i, \infty) \). The strong type’s (IC) is: \( b_w - c_s^P \hat{l}^P_i \geq (1 - p) - c_s^P \). The weak type’s (IC) is: \( b_w - c_s^P \hat{l}^P_i \geq 0 \). Both (IC) are satisfied whenever \( \hat{l}^P_i \leq \min \{\frac{\mu^P(c_s^P(b_w), b_w^P)}{c_s^P}, \frac{\mu^P(c_s^P, b_w)}{b_w} \} \). Consequently, when \( c_s^P > 1 - p - b_w \), any signaling strategy satisfying \( \zeta^P(s) = \zeta^P(w) = (m, l^P_i) \), with \( l^P_i \leq \min \{\frac{\mu^P(c_s^P(b_w), b_w^P)}{c_s^P}, \frac{\mu^P(c_s^P, b_w)}{b_w} \} \) can be part

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21By Assumptions 1 and 2 \( d^*(\zeta^P, \zeta^A, b_w, 1) = 2 \) for all \( \zeta^P \) (since \( c_s^P > (1-p)b_w \)). This directly implies that it is never a best response for the decision-maker to propose \( b < b_w \). Suppose there exists \( \zeta^P \) such that \( b^*(\zeta^P, \zeta^A) = 1 \). It must be that \( \mu^P(\zeta^P) \) satisfies \( \mu^P(\zeta^P)(1-p-k/2) \geq b_w \), which contradicts \( 1-p-b_w < k/2 \).
of a pooling equilibrium.
Since \( b^*(\zeta^P, \zeta^A) = b_w \) and \( d^*(\zeta^P, \zeta^A, b_w, 0) = 0, d^*(\zeta^P, \zeta^A, b_w, 1) = 2 \) in a pooling equilibrium, we directly obtain \( l^P_o(\tau) = 0, \tau \in \{s, w\} \).

I now prove the results regarding the anti-change SIG influence (denoting \( \zeta^P \) the pro-change SIG’s uninformative signal). As \( q^P = 0 \) and \( (1 - p) < c_w^P \), \( d(\cdot) = 1 \) is a strictly dominated strategy. I focus on the effect of anti-change SIG’s inside lobbying strategy on policy choices with all players playing their best response down the game tree.

**Lemma 6.** The anti-change SIG plays a separating strategy (i.e., \( \zeta^A(s) \neq \zeta^A(w) \)) on the equilibrium path only if \( b(\zeta^P, \zeta^A(s)) < b(\zeta^P, \zeta^A(w)) \).

**Proof.** First, notice that since \( (1 - p)b_w - k > 0 \) (so \( d(\zeta^P, \zeta^A, b_w, 1) = 2 \) is a best response) and \( b_w > 1 - p - k \), the decision-maker’s best response after observing \( \zeta^P(w) \) is \( b(\zeta^P, \zeta^A(w)) = b_w \). The rest of the proof proceeds by contradiction. Suppose \( b(\zeta^P, \zeta^A(s)) > b(\zeta^P, \zeta^A(w)) = b_w \). The strong type’s (IC) is: \( -(1 - p)b(\zeta^P, \zeta^A(s)) - c^A - c^A_{\zeta^A}(s) \geq -(1 - p)b_w - c^A - c^A_{\zeta^A}(w) \) as \( l^A_o(b_w, \zeta^P; s) = 1 \).

The weak type’s (IC) is: \( -b_w - c^A_{\zeta^A}(w) \geq -(1 - p)b(\zeta^P, \zeta^A(s)) - c^A - c^A_{\zeta^A}(s) \). Using the definition of \( b_w \), the (IC) are equivalent to: \( c^A_{w}(l^A_i(w) - l^A_i(s)) \leq (1 - p)(b(\zeta^P, \zeta^A(s)) - b_w) \leq c^A_{w}(l^A_i(w) - l^A_i(s)) \).

The first inequality implies \( l^A_i(w) > l^A_i(s) \). Since \( c^A_{w} > c^A_{s} \), we have reached a contradiction.

**Lemma 7.** The anti-change SIG plays a separating strategy on the equilibrium path only if: \( l^A_i(w) = 0 \) and \( l^A_i(s) > 0 \).

**Proof.** By Lemma 6 (i.e., \( b(\zeta^P, \zeta^A(s)) < b_w \)), a weak type’s (IC) is satisfied only if \( l^A_i(s) > 0 \). \( l^A_i(s) = 0 \) follows by the Intuitive Criterion.

**Proof of Lemma 6.** Necessity. Suppose \( \zeta^A(s) = (s, l^A_i(s)) \neq \zeta^A(w) = (w, l^A_i(w)) \). The decision-maker’s best response is \( b(\zeta^P, \zeta^A(s)) = 1 \) when \( 1 - p - k > b_s \) so the decision-maker strictly prefers 1 to \( b_s \) or \( (1 - p)b_s - k < 0 \). To see this last result, notice that when the inequality holds, the anti-change SIG’s outside lobbying best response is \( l^A_o(b_s, \zeta^P; s) = 1 \) since \( d(\zeta^P, \zeta^A(s), b_s, 1) = 0 \) so the decision-maker gets 0 by choosing \( b_s \), \( (1 - p)b_w - k \) by choosing \( b_w \) and \( (1 - p) - k \) by choosing \( b = 1 \). By Lemma 6, a separating equilibrium cannot exist when \( b_s < \max\{1 - p - k, \frac{k}{1 - p}\} \).

Assume \( b_s \geq \max\{1 - p - k, \frac{k}{1 - p}\} \) so \( b(\zeta^P, \zeta^A(s)) = b_s \). Using Lemma 7, the reasoning in the text implies the weak type’s (IC) is: \( -b_w \geq -b_s - c^A_{\zeta^A}(s) \). By the Intuitive Criterion, \( l^A_i(s) = \frac{b_w - b_s}{c^A_{\zeta^A}} \)
the equilibrium path. It can be checked that a pooling equilibrium exists for all other parameter
signal $q$. When the decision-maker chooses $b = 1$, her expected utility is:

$$
\mu^A(\zeta^A) = q^A.
$$

When the decision-maker chooses $b = 1$, her expected utility is $1 - p - k$; when $b = b_w$, her expected utility is:

$$
q^A[(1 - p)b_w - k] + (1 - q^A)b_w \quad \text{(as $l_{0}^A(b_w, \zeta^P; s) = 1$ and $l_{0}^A(b_w, \zeta^P; w) = 0$)};
$$

when $b = b_s$, her expected utility is $b_s$ if $(1 - p)b_s - k \geq 0$ and 0 otherwise. When $b_s < \max\{1 - p - k, k/(1 - p)\}$,

$b = b_s$ is strictly dominated by $b = 1$. Simple computations yield the decision-maker’s best response
is $b(\zeta^A, \zeta^P) = b_w$ if and only if $q^A \leq q^A \equiv \frac{b_w - (1 - p - k)}{p b_w + k}$. When $b_s > \max\{1 - p\}b_w, k/(1 - p)\}$,

$b = b_s$ is strictly dominated by $b = b_s$. Simple algebra yields $b(\zeta^P, \zeta^A) = b_w$, $\zeta^A$ if and only if:

$$
q^A \leq q^A(b_s) \equiv \frac{b_w - b_s}{p b_w + k}.
$$

A pooling equilibrium does not exist when $b_s > \max\{1 - p\}b_w, k/(1 - p)\}$ and $q^A < q^A(b_s)$. Consider
the out-of-equilibrium signal $\hat{\zeta^A}(s) = (s, l_{0}^A(b_s))$ and suppose decision-maker’s out-of-equilibrium
belief satisfies: $\mu^A(\hat{\zeta^A}(s)) = 1$ so $b(\zeta^P, \zeta^A(s)) = b_s$. Only a strong type has an incentive to send
signal $\hat{\zeta^A}(s)$. By the Intuitive Criterion, the anti-change SIG does not play a pooling strategy on
the equilibrium path. It can be checked that a pooling equilibrium exists for all other parameter
values.

Proof of Proposition 4. Point 1. Suppose $q^A \leq q^A$. Consider the following belief structure: $\mu^A(\zeta^A) = 1$
when $\zeta^A = (m, l_{1}^A)$ for $m \in \{s, w\}$ and $l_{1}^A \in [0, \hat{l}_{1}^A)$ with $\hat{l}_{1}^A > 0$, and $\mu^A(\zeta^A) = q^A$, otherwise.

Given this belief structure, the decision-maker’s best response is: $(b^∗(\zeta^P, \zeta^A) = 1, d^∗(\zeta^P, \zeta^A, 1, 0) =
0, d^∗(\zeta^P, \zeta^A, 1, 1) = 2), \forall \zeta^A \in \{s, w\} \times [0, \hat{l}_{1}^A)$ and $(b^∗(\zeta^P, \zeta^A) = b_w, d^∗(\zeta^P, \zeta^A, b_w, 0) = 0, d^∗(\zeta^P, \zeta^A, b_w, 1) =
2), \forall \zeta^A \in \{s, w\} \times [\hat{l}_{1}^A, \infty)$. The strong type’s (IC) is:

$$
-(1 - p)b_w - c_w - c_w l_{1}^A \geq -(1 - p) - c_w.
$$

The weak type’s (IC) is: $-b_w - c_w l_{1}^A \geq -(1 - p) - c_w$. Both (IC) are satisfied whenever $\hat{l}_{1}^A \leq \frac{(1 - p)(1 - b_w)}{c_w} \equiv \hat{l}_{1}^A$. So any signaling strategy satisfying $\zeta^A = (m, l_{1}^A)$ with $l_{1}^A \leq \frac{(1 - p)(1 - b_w)}{c_w}$ can be part of a pooling equilibrium. For outside lobbying, notice that $l_{0}^w(b_w, \zeta^P; s) = 1$ and $l_{0}^w(b_w, \zeta^P; w) = 0$. Suppose
\( q^A > q^A \) so \( b^*(\zeta^A, \zeta^P) = 1 \) for all \( \zeta^A \) and \( l_{i}^{A*} = 0 \). Obviously, \( l_{o}^{A*}(1, \zeta^P; \tau) = 1 \) for all \( \tau \in \{s, w\} \).

**Point 2.** Similar logic as point 1.
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