Title: Herglotz and Caratheodory over the ages

Abstract:

Herglotz's representation of holomorphic functions with positive real part and Caratheodory's theorem on approximation by inner functions are two well-known classical results in the theory of holomorphic functions on the unit disc. In this talk, we shall first see that they are equivalent. On a multi-connected domain Ω, a version of Heglotz's representation is known. Caratheodory's approximation was not known. This will be formulated. We then show that it is equivalent to the known form of Herglotz's representation. Additionally, it also enables us to prove a new Heglotz's representation in the style of Koranyi and Pukanszky. Of particular interest is the fact that the scaling technique of the disc is replaced by Caratheodory's approximation theorem while proving this new form of Herglotz's representation. Caratheodory's approximation theorem is also proved for matrix-valued functions on a multi-connected domain. The part of this talk which is new is joint work with Poornendu Kumar and Mainak Bhowmik.