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Title: Exploring our capacities

Abstract:

The talk will survey the use of capacities, in the hope of drawing them to the attention of young researchers. A capacity assigns a nonnegative number to each set.

In the mathematical toolbox, the capacities are the very sharp, handheld tools. Many problems are not amenable to 'soft' methods, and require the use of a suitable capacity. Often, all you have to do is (1) define the capacity and (2) understand it (well enough). Each of these steps may present more or fewer difficulties. So the two perennial questions are: (1) have we the capacity needed to deal with our problem, and (2) do we understand our capacities?

Types of capacities, apart from measures, include Hausdorff contents, kernel capacities, iterated potential capacities, extremal lengths, condenser capacities, and intrinsically-defined capacities such as L-F capacities, where L is an elliptic operator and F is a function space.

Properties a capacity might have include kinds of subadditivity, translation-invariance, affine-invariance, being inner or outer, density properties, kinds of instability, continuous dependence on a parameter, and regularity on Borel or Suslin sets.

Capacities arise in connection with the Dirichlet problem, partial differential equations, removable singularities, boundary behaviour, inner functions, biholomorphic classification, peak points, point evaluations and point derivations, approximation problems, singular integral operators, pseudodifferential operators, function spaces and spaces of distributions, value distribution, convergence of Fourier series, Brownian motion, fine topologies, and various kinds of exceptional sets.

There is a large body of results, and there are many open questions. Over a third of my own papers (obtainable at https://www.logicpress.ie/aof/publications.html) are connected to capacities of one kind or another, and many leave questions to be explored.