Professor Nige Ray
University of Manchester

Octonions and projective space: geometry without associativity

Abstract

In this talk I shall discuss the sequence of number systems that begins with the real numbers, the complex numbers, and the quaternions, and introduce the next term, namely the octonions. In passing, I shall mention a little of their history, which is rather complicated (and, of course, features the Royal Canal!) Then I shall describe the associated projective line and projective plane in terms of 3 x 3 Hermitian matrices, which are required to deal with the lack of associativity. The set of these matrices has a particularly rich geometrical structure, including its group of symmetries F_4, which appears in certain aspects of theoretical physics. In passing, I shall touch upon linking numbers, homotopy theory, and perhaps even cohomological invariants. My aims are to provide an overview, rather than technical details, and to make the talk accessible and entertaining for a general mathematical audience.