Groups that are isoclinic to rings

Let F be a finite algebraic system F in which multiplication is denoted by juxtaposition. The *commuting probability of* F is

$$\mathrm{Pr}(F) = \frac{|\{(x,y) \in F \times F \ : \ xy = yx\}|}{|F|^2},$$

where $|\cdot|$ denotes cardinality. Much has been much written on $\mathrm{Pr}(G)$ where G is a finite group. Together with MacHale and Ní Shé, we previously studied $\mathrm{Pr}(R)$ where R is a finite ring, making use of an associated notion of ring isoclinism.

In this talk, we compare and contrast the values attained by $\mathrm{Pr}(G)$ and $\mathrm{Pr}(R)$ as G and R range over certain classes of groups and rings, respectively. We show in particular that the set of values that arise for finite rings and for finite class-2 nilpotent groups are the same. Proving this involves the consideration of certain triples $T = (A, B, k)$ associated with both class-2 groups and rings. Isomorphism of such triples generalizes the previous notions of group isoclinism and ring isoclinism, and commuting probability of groups and rings is an isomorphic invariant of the associated triples.

This talk is based on joint work with Des MacHale.