Title

Real elements of odd order and real characters of finite groups

Abstract

The (irreducible) complex characters of a finite group are complex functions on the group which encode its complex representations. It is known that the number of characters equals the number of conjugacy classes and the number of real-valued characters equals the number of real conjugacy classes. Now each real character is either orthogonal or symplectic, as distinguished by its Frobenius-Schur indicator. It is a long-standing and significant open problem to determine the number of each type in group-theoretic terms. In this talk we discuss an approach to this problem, developed by us in conjunction with R. Gow and others, which relates the problem to the conjugacy classes of elements of odd order in the group. In particular we make use of the Brauer characters and the principal indecomposable (pi-) characters of the group. In a new result we show that the number of orthogonal pi-characters coincides with the number of strongly real classes of odd order elements. Here strongly real means inverted by an involution. We speculate that the number of symplectic complex characters is at least the number of weakly real conjugacy classes of odd order elements.

This talk is aimed at non-experts in mathematics and related disciplines. In particular we will attempt to explain the essentials of all important definitions and results as we proceed.