

Title: Gauss factorials, and Gauss primes (joint work with Karl Dilcher)

Abstract:

The notion of a Gauss factorial was introduced in a 2008 paper by Karl Dilcher and the speaker. Subsequently this entirely elementary object enabled us to vastly extend a classic theorem of Gauss, his remarkable and celebrated 'binomial coefficient congruence' (1828).

Then, in a later paper, we introduced the notion of a Gauss prime (it relates to the Gauss binomial congruence), which is naturally and exclusively related to a very particular Gauss factorial congruence (let's call it cong_1), the least solutions of which are $n_1 = 205479813$, $n_2 = 1849318317$ and $n_3 = 233456083377$.

The next known solution is the 155-digit $N_1 =$

1830203294953468474759429106413

1327343388101729948250251743238

7652854771562362994415635012597

9683169277996381731347880880679

9687477000415482891357317727809

(there are larger known ones ...)

What do we know about the solutions to cong_1 ? We know **all** of them (in a sense to be explained). Are there solutions between n_3 and N_1 ? Possibly. Why don't we know one way or the other? How can one say that we know all solutions to cong_1 , and not know if there are any between n_3 and N_1 ? These are the kinds of issues that I intend elucidating.

Should time allow I will make a brief mention of another similar (but more elusive) type of prime which we also introduced, a 'Jacobi' prime.

This talk will be at a completely elementary level, understandable even by an undergraduate who knows only the meaning of a congruence ($a = b \pmod{n}$).

REFERENCES.

1. The Gauss-Wilson theorem for quarter-intervals, Acta Mathematica Hungarica, Vol. 142 (2014), no. 1, 199–230.
2. A role for generalized Fermat numbers, Math. Comp. 86 (2017), 899-933.