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Title: Periodic rings, Boolean numbers, and related questions

Abstract: We say that a ring R is periodic, and has period n, if it satisfies the identity $x^{n+1} = x$. Sometimes a period forces a smaller period: for instance, period 5 forces period 1 in the sense that a ring with period 5 necessarily has period 1 (i.e. it is Boolean). Let $j(n)$ be the smallest period forced by period n. We call n Boolean if $j(n) = 1$.

We discuss periodic rings using Jacobson's structure theory of rings, leading us to the concept of a J-divisor of a positive integer. Using J-divisors, we can find a simple formula for $j(n)$. We investigate the set of Boolean numbers and more generally the set of n such that $j(n) = k$ for fixed k: we show that either this equation has no solution or its set of solutions consists of certain multiples mk of k, where the set of allowable m depends on k but always includes infinitely many prime and infinitely many composite numbers. Finally, we discuss some striking patterns that we do not yet understand.