Title: On semi-free actions whose orbit spaces are manifolds

Abstract:

Semi-free S^1 actions on spheres have been studied in some detail by Montgomery, Yang and Levine, among others. One interesting case of such actions is when the fixed point set has codimension 4; in this case the orbit space admits a smooth manifold structure. For instance, all exotic 7-spheres admit infinitely many such actions with quotient space $\mathbf{S}^6$; the actions are classified by the knotting of the fixed point set, $\mathbf{S}^3$. In this talk we will explore which 5-manifolds admit such actions with quotient a simply connected 4-manifold. We will show:

Suppose M^5 is (i) a simply connected 5-manifold admitting a semi-free S^1 action with (ii) codimension-4 fixed point set with n components and (iii) orbit space (manifold) M^* with $b_2(M^*) = k$. Then M is a connected sum of $n + k - 1$ copies of $\mathbf{S}^3$-bundles over $\mathbf{S}^2$. Moreover, M is spin if and only if M^* is spin.

Time permitting we will also talk about semi-free S^3 actions on 8-manifolds and a generalization of this type of action with manifold quotient. This is joint work, in progress, with John Harvey and Krishnan Shankar.