A characterisation of the Gaussian Lipschitz space

Peter Sjögren
University of Gothenburg

Abstract

In $\mathbb{R}^n$ we replace Lebesgue measure by a Gaussian measure and the Laplacian by the Ornstein-Uhlenbeck operator $\Delta - 2x \cdot \text{grad}$. This operator is self-adjoint in the Gaussian L^2 space. In this setting, a Lipschitz space was defined by Gatto and Urbina, in terms of the gradient of the Ornstein-Uhlenbeck Poisson integral of the function. We show that this space can also be described as a Lipschitz space in the ordinary sense, by means of an inequality for the modulus of continuity. The proof is based on several sharp estimates for the Ornstein-Uhlenbeck Poisson kernel and its gradient, also of independent interest.

This is joint work with Liguang Liu (Beijing).