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Title: Quasihyperbolic geodesics are hyperbolic quasigeodesics

Abstract:

The hyperbolic metric, known also as the Poincaré metric for simply connected domains, is an important tool in complex analysis and complex geometric function theory.

The quasihyperbolic metric is a key tool in quasiconformal analysis, and has many other applications, for instance Peter Jones' classification of BMO extension domains.

Both of these metrics are defined in the setting of hyperbolic plane domains (plane domains whose complement include at least two points), but they are not in general bilipschitz equivalent. However, we prove that a geodesic curve for either one of them is in a certain precise sense not far from being a geodesic curve for the other, regardless of the domain. More precisely, we prove that hyperbolic and quasihyperbolic quasigeodesics are quantitatively the same curves (with no quantitative dependence on the domain).

We also show that a domain is Gromov hyperbolic with respect to one of these metrics if and only if it is Gromov hyperbolic with respect to the other.