

Stephen McGuire

How Small Can Polynomials be in an Interval of Given Length?

Abstract

I will discuss two extensions to a result of V. I. Bernik from 1983. Bernik's result was a key tool for obtaining results in many counting problems over the past 35 years but it had restrictions. The first extension I provide removes some of these restrictions. Furthermore, as an application of this, a new result on the distribution of conjugate algebraic points in a 3-dimensional cube was obtained. This result was obtained with Dr. Detta Dickinson and Dr. Natalia Budarina. Although it was this problem that motivated the interest into trying to extend the result of Bernik there are many more questions that can now be tackled using the extension.

The second extension to the result of Bernik allows one to move from the original statement, which only dealt with real intervals, to a statement that now deals with the space $\mathbb{R} \times \mathbb{C} \times \mathbb{Q}_p$. Although this was not the first extension of this kind, the method of proof I produce leads to a very useful proposition. This proposition allows many questions previously considered to be looked at again. One such open question on the size of the set of polynomials with bounded discriminant will be shown to be a simple application of this proposition.