IS THERE SUCH A THING AS A LONELY RUNNER?

Let $n \in \mathbb{N}$ and consider n runners on a circular track of unit length. A runner is called **lonely** at a time t if all other runners are a distance of at least $\frac{1}{n}$ from that runner at this time t. The lonely runner conjecture asserts that, for each runner, there exists a time t at which that runner becomes lonely. Certainly if any two runners share the same speed no such time t can exist, so it can always be assumed that the runners have distinct speeds.

In its most simplistic form the lonely runner conjecture is the following open problem:

Conjecture.
Let $n \in \mathbb{N}$ and consider n runners on a circular track of unit length. At time $t = 0$ all runners start from the same position with distinct fixed speeds. Then each runner becomes lonely at some time t.

The conjecture originated from questions in the area of Diophantine approximation due to J. M. Wills [3] in 1967 and in the area of view-obstruction problems in geometry due to T. W. Cusick [2] in 1973. The conjecture also has connections to questions about chromatic numbers of distance graphs [4], [5] and to flows in regular matroids [1]. Even though it has been over fifty years since the lonely runner conjecture was first stated, it remains open. Furthermore, it is only know to hold for seven or less runners.

The aim of the talk is to give a general survey of the lonely runner conjecture. In particular, we will discuss what results there are to date and how the conjecture appears in many different forms throughout mathematics.

References

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