

The Asymmetric Effects of Uncertainty Shocks

Draft*

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Abstract

We investigate the effects of financial uncertainty shocks on macroeconomic aggregates in the United States. Financial uncertainty is captured by appealing to some indicators recently developed. Relying on a nonlinear VAR, we isolate the effects of uncertainty in both recessionary and expansionary periods. As main findings, we report how uncertainty shocks have a negative macroeconomic impact across the business cycle. The asymmetric effects are persistent when we include the recent recession. To reduce the fall in macroeconomic variables, the "unconventional" monetary policy plays an important role.

JEL classification: C50, E32, E52

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1 Introduction

Recent empirical studies provide evidence how uncertainty is one of the main driver of the global financial crisis and the subsequent slow recovery. In particular, researchers show that a contraction in the business cycle is reported when an unexpected increase in uncertainty features. There are several measures which could be adopted to proxy uncertainty. Bloom (2009) has pioneered the use of the VIX, the implied stock market volatility based on S&P index, and Baker, Bloom, and Davis (2016) propose an Economic Policy Uncertainty (EPU) index based on news article counts. Moreover, Rossi and Sekhposyan (2015) rely on the Survey Professional Forecasters to construct a forecasting index. Last but not least, Jurado, Ludvigson, and Ng (2015) develop a macroeconomic uncertainty index based on the common variation in forecast error of a large number of economic indicators. Meanwhile, Ludvigson, Ma, and Ng (2017) build two different measures for macroeconomics and financial uncertainty. In this project, we decide to proxy the uncertainty using the financial uncertainty measure as discussed in Ludvigson, Ma, and Ng (2017). In this way, we can study the impact on the macroeconomic variables and on the monetary policy of the uncertainty through financial channel. Looking at the monetary policy stance, Central Banks are used to offset the negative macroeconomic effects of uncertainty shocks lowering the interest rate. However, when the monetary policy rate is closed to the Zero Lower Bound (ZLB), as occurred in the Great Recession, further stimulus is needed, and an uncertainty shock may push the Central Bank to rely on "unconventional" policy measures.

How do uncertainty shocks affect the conventional and unconventional monetary policies conducted by the Federal Reserves?

We answer this research question contributing the literature by proposing a nonlinear estimation to understand the asymmetric responses of the macroeco-

economic variables to a financial uncertainty measure, focusing on the role of the U.S. monetary policies before and after the Great Recession. Researchers have already highlighted the role of nonlinearities in the transition of uncertainty shocks on macroeconomic activities across the business cycle or at the Zero Lower Bound (i.e., Enders and Jones, 2013; Bijsterbosch and Guérin, 2013; Caggiano, Castelnuovo, and Groshenny, 2014; Caggiano, Castelnuovo, and Nodari, 2015; Alessandri and Muntaz, 2014; Caggiano, Castelnuovo, and Pellegrino, 2015; Popp and Zhang, 2015). But, as far as we know, this paper is the first one stressing the importance of the different monetary policies ("conventional" and "unconventional") in expansion and recession periods.

To empirically scrutinize the potential asymmetric effects of macroeconomic uncertainty shocks, we model a set of macroeconomic indicators with a Smooth Transition Vector Auto Regression (STVAR). This approach allows us to estimate the effects of uncertainty shocks conditional on the state of economy (i.e., expansions versus recessions). To model the endogeneity of the transition from a state to another after an uncertainty shock occurs, we compute the Generalized Impulse Response Functions (GIRFs) proposed by Koop, Pesaran, and Potter (1996). Since the GIRFs depend on the initial condition, we study the evolution of the GIRFs over histories (i.e., recessions and expansions). This allows us to compare the IRFs in "good times" versus "bad times". We estimate a vector of endogenous variables very close to the one proposed by Christiano, Eichenbaum, and Evans (2005) and by Jurado, Ludvigson, and Ng (2015). The STVAR includes real variables such as Industrial Production (IP), Employment (Empl), Inflation (CPI), monetary tools, and uncertainty measures. Following Gambacorta, Hofmann, and Peersman (2014), we proxy the "unconventional" monetary policy with the total asset of Fed's Balance Sheet (BS) which represents the quantitative easing. Since such measure is downloadable from the

Federal Reserve of St. Louis only from 2002, we collect monthly data of the total asset of Fed's Balance Sheet for the remaining period (1985-2001). This proxy allows us to capture the "unconventional" monetary policy effectiveness during the Zero Lower Bound period.

Our main findings show that one-standard deviation uncertainty shock triggers asymmetric effects on macroeconomic aggregates across the business cycle. Uncertainty shock has contractionary effects both in recessions than in expansions, but such effects are more pronounced in the recessionary periods. To offset the macroeconomic fluctuation, the Federal Reserve reacts via both the "conventional" (decreasing the Federal Fund rate) and the "unconventional" monetary policies (increasing the assets of the Fed's balance sheet by 20%). The reaction of FFR is found to be quantitatively smaller in expansions than in recessions. The reaction of the Federal Reserve to financial uncertainty shock via the "unconventional" monetary policy is long-lasting. Overall the monetary policy easing associated with a contraction of economic activity is consistent with an inflation-targeting strategy pursued by the monetary policymakers.

Our results provide new evidence on the role played by uncertainty shocks on "unconventional" monetary policies. A battery of robustness checks confirms our main findings. From a policy standpoint, this evidence helps the policymaker to implement tailored monetary policy instruments across the business cycle and in particular when ZLB is binding.

Counterfactual experiments investigate about the role played by the monetary policies in recessionary periods. In particular, we assess the importance of the "unconventional" monetary policy represented by the Balance Sheet. To complete the analysis about monetary policies, we estimate our baseline model introducing a new monetary policy tool, the Shadow Short Rate (SSR). Several SSRs have been developed to explain the negative behaviour of the short term

interest rates relying on the term-structure (as in Krippner, 2013 and Wu and Xia, 2016) and on macroeconomic and monetary variables (as in Lombardi and Zhu, 2014). As surveyed in Comunale and Striaukas (2017), researchers can use Shadow Short Rates to produce a summary metric for the stance of the "conventional" and "unconventional" monetary policies.

The remainder of the paper is organized as follows. Section 2 introduces the estimated model, the Smooth Transition VAR and the data. Section 3 documents the empirical results of the baseline model. Section 4 discusses the asymmetric responses in expansionary and recessionary periods. Section 5 illustrates several robustness checks and Section 6 shows the counterfactual experiments. Section 7 summarizes the findings and provides concluding remarks.

2 Data and Methodology: A Smooth Transition VAR

The estimated Smooth-Transition VAR model (STVAR) is defined as follows:

$$X_t = F(z_t)\Pi_R(L)X_t + (1 - F(z_t))\Pi_E(L)X_t + \varepsilon_t, \quad (1)$$

$$\varepsilon_t \sim N(0, \Omega_t), \quad (2)$$

$$\Omega_t = F(z_t)\Omega_R + (1 - F(z_t))\Omega_E, \quad (3)$$

$$F(z_t) = \exp(-\gamma z_t)/(1 + \exp(-\gamma z_t)), \gamma > 0, z_t \sim N(0, 1). \quad (4)$$

where X_t is a set of endogenous variables, $\Pi(L)_R$ and $\Pi(L)_E$ are the polynomial matrices in the lag operator L capturing the dynamics of the system during recessions and expansions, respectively. The vector of reduced-form residuals (ε_t) has zero-mean and heteroskedastic, state-contingent variance-covariance matrix

Ω_t , where Ω_R and Ω_E refer to the covariance structure of the residuals in recessions and expansions, respectively. $F(z_t)$ is a logistic and continuous function bounded between zero and one which depends on the state variable z_t . The slope parameter γ dictates how smooth is the transition from one regime to another, i.e. from recessions to expansions and *vice versa*. If $\gamma \rightarrow \infty$ in (4), then the transition from one state of economy to the other one is abrupt. Conversely, small value of γ implies that such transition is smooth.

The vector of endogenous variables relies on $X_t = [X_{1t} X_{2t} X_{3t}]'$, where X_{1t} includes the Inflation (CPI), the Industrial Production (IP), and the Employment (Empl). The X_{2t} incorporates proxies for the conventional and unconventional monetary policy measure, the federal fund rate and the total assets of the Fed's balance sheet, respectively. Following Gambacorta, Hofmann, and Peersman (2014), we proxy the unconventional monetary policies via the total assets of Fed's Balance Sheet.¹ The vector X_{3t} includes our proxy of uncertainty. We rely on recent measures of financial uncertainty proposed by Ludvigson, Ma, and Ng (2017) based on the common variation in the h-steps-ahead forecast errors of a large number of financial indicators, $u(h)$. We use the uncertainty measure with forecast horizon at 1-month ($u01$), and we perform robustness checks relying on the 3-months ($u03$) and 12-months ($u12$) financial uncertainty indicators. Figure 1 plots the uncertainty measures versus the business cycle turning points (shades

¹The Federal Reserve faced the Great Recession by adopting an extraordinarily expansionary monetary policy stance, lowering policy rates close to zero to stimulate the economy. However, with monetary policy rates close to the zero lower bound (ZLB), when further stimulus was needed, Central Banks turned to non-interest rate, or non-standard, policy measures. The measures adopted by Central Banks to counteract deflationary pressures and to foster economic growth included increased liquidity provision, extending the term of lending, modifying the collateral framework, forward guidance, and asset-purchase programs (i.e., quantitative easing, QE). The aims of these programs have been to reduce long-term interest rates and thereby stimulate the economy. Such stimulus has substantial effects on the size of central banks' Balance Sheets (Colombo, 2015). Meaning and Zhu (2011) use the Federal Reserve Balance Sheet information to proxy the unconventional monetary policy tools. Peersman (2011) studies the (linear) macroeconomic effects of unconventional monetary policy in the Euro Area relying on the size of ECB's Balance Sheet. Also, Gambacorta, Hofmann, and Peersman (2014) focus on the total assets of Central Banks' Balance Sheet to proxy unconventional monetary policies.

area). Notice that financial uncertainty spikes occur during recessionary periods.

The uncertainty shock is identified via the Cholesky-decomposition, with the sample assumptions provided in Christiano, Eichenbaum, and Evans (2005) and widely adopted in the monetary policy VARs of the literature. In other words, the slow moving variables (CPI, Industrial Production, and Employment) are ordered first, whereas the fast moving variables (monetary policy tools) are ordered last. This ordering implies that monetary policies depend on the real activities. In setting the monetary policy tools in vector X_2 , we place the Fed total asset after the FFR. This reflects the idea that Fed relies on unconventional monetary policies by expanding its Balance Sheets after the FFR approaches to zero. The uncertainty measure is set last in vector X_t . It means that we "purge" our uncertainty indicator from the contemporaneous movements of our macroeconomic variables, therefore sharpening the identification of uncertainty shocks. This identification implies that macroeconomic variables react to uncertainty shocks with a lag.² This assumption is plausible for monthly estimations and is in line with Jurado, Ludvigson, and Ng (2015). However, our identification scheme differs from that of Jurado, Ludvigson, and Ng (2015) since we take into account the Fed policy implementation during the Great Recessions.

All the variables in X_t enter in natural logarithms and in real terms (except the interest rate).

The transition variable z_t and the calibration of the smoothing parameter γ are justified as follows. As developed in Auerbach and Gorodnichenko (2012) and Caggiano, Castelnuovo, and Groshenny (2014), we employ a standardized backward-looking of twelve-month moving average of industrial production growth.³ We calibrate the smoothness parameter γ to match the prob-

²The main results are not affected when the uncertainty index is set first in vector X_t . The results are available upon request.

³The transition variable z_t has been standardized to be comparable to those employed in the literature.

ability of being in recessions as identified by the NBER business cycle dates (15% in our sample). The recessionary phase is defined as a period in which $\Pr(F(z_t) \geq 0.85) \approx 15\%$. It means that the economy spends about 15% of time in recessions and 85% in expansions. This implies setting $\gamma = 1.8$. The choice is consistent with the threshold value $\bar{z} = -0.9\%$ discriminating recessions and expansions. In particular, if the realizations of the standardized transition variable z_t is lower (higher) than the threshold value \bar{z} , it will be associated to recessions (expansions). Figure 2 plots the transition function $F(z_t)$ versus the NBER turning points and shows that high values of $F(z_t)$ tend to be associated with NBER recessions.

Given the high nonlinearity of the model, we estimate the STVAR in (1) relying on Markov-Chain Monte Carlo simulation (Chernozhukov and Hong, 2014), see section B of the Appendix for details. To model the endogeneity of the transition from one state to another after an uncertainty shock occurs, we compute the Generalized Impulse Response Functions (GIRFs) proposed by Koop, Pesaran, and Potter (1996). Since the GIRFs depend on the initial condition, we study the evolution of the GIRFs over histories (i.e., recessions and expansions). This allows us to compare IRFs in normal times versus uncertainty times. Our data are monthly and spans from 1985M1 through 2011M12. We estimate a nonlinear VAR including five lags, as suggested by the Akaike information criterion. Our model includes a constant. The data are seasonally adjusted and retrieved from the Federal Reserve Bank of St.Louis.

Before estimating the STVAR in (1), we perform a linearity test. Linearity is tested replacing the transition variable (z_t) by the third order Taylor series approximation around $\gamma = 0$, as suggested by Teräsvirta and Yang (2014). We perform an LM test which suggests a strong rejection of the linearity for the system as a whole in favor of a STVAR.

3 Results

Figure 3 and Figure 4 plot the Generalised IRFs (GIRFs) to a one-standard deviation uncertainty shock identified via the Ludvigson, Ma, and Ng (2017) financial uncertainty measure with forecast horizon equal to 1-month (u01). The dotted-blue lines denote the GIRFs in expansions, whereas the red lines the ones in recessions. The shaded bands refer to the 68% confidence intervals. The impulse responses are interpreted as deviations from the steady-state and expressed in percentage change. Figure 3 reports the 68% confidence intervals for the expansionary responses, while Figure 4 reports the 68% confidence intervals for the recessionary ones. At the first glance, uncertainty shocks trigger negative macroeconomic fluctuations both in expansions and in recessions. However, in expansions the reactions of macroeconomic variables to uncertainty shocks are quantitatively smaller than in recessions.

In recessions, an uncertainty shock decreases the Industrial Production by -1% after two months the shock occurs. Hence, the response, which is statistically significant as denoted in Figure 3, hits a trough of -6.25% at 8 months after the shock occurs. Afterwards, the response returns slowly to its steady-state which is not completely reached after 60 months. Moreover, the shock has a long-lasting deflationary effect. The inflation reaches a trough of -2.9% (the 60th month), and it remains below the pre-level shock for all the period. The shock decreases employment by -2.36% thirteen-months after the shock occurs. Interestingly, the GIRFs predict a strong reaction of the Federal Reserve via "unconventional" monetary policies which determine an increase in the Fed total assets of 20% with respect to the pre-shock levels. The reaction of the Central Bank via "conventional" monetary policy tools (decreasing the short term interest rate) is larger in recessions (as the Great Recession) than in expansions. This results may be driven by the fact that our sample size includes the period

in which the FFR approaches zero.

In expansions, the Industrial Production decreases with a trough response of -1.9% at fifteen-months after the shock occurs and after that, the effect gradually returns to the steady-state. The response of the other macroeconomic variables, Inflation (CPI) and Employment (Empl) is qualitatively similar to the Industrial Production one and their troughs coincide with that of Industrial Production. The short term interest rate (FFR) reacts to uncertainty shock via the "conventional" monetary policy, following an inflation targeting strategy path.

These results corroborate those reported in previous contributions on the "demand" type of effects triggered by uncertainty shocks in the U.S. economy (i.e., Bloom, 2009; Baker, Bloom, and Davis, 2013; Caggiano, Castelnuovo, and Groshenny, 2014; Leduc and Liu, 2013; Colombo, 2013; Alessandri and Muntaz, 2014). Our findings are supported by the theoretical studies (i.e., Basu and Bundick, 2015; Basu and Bundick, 2015) which document the fall of nominal and real variables after an uncertainty shock occurs, and by the empirical analysis in which uncertainty shocks is found to trigger asymmetric effects across the business cycle (i.e., Caggiano, Castelnuovo, and Pellegrino, 2015; Caggiano, Castelnuovo, and Groshenny, 2014). Moreover, our evidences are in line with the previous studies which highlight that when the investment irreversibility, the level of uncertainty affects the value of investment opportunities (i.e., Bernanke, 1983; Bloom, 2009; Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry, 2014).

4 Asymmetric reactions across regimes and specifications

As main findings, we provide evidence how uncertainty shock has a recessionary behavior in both expansions and recessions. In particular, we contribute to the literature showing how larger (in absolute value) negative effects in recessions than in expansions. *Are the reactions of macroeconomic variables to uncertainty shocks statistically significant different across regimes?* We answer this question proposing a statistical test based on the empirical density of the difference between the reaction of macroeconomic variables across regimes. The empirical density is based on 500 realizations of such differences for each horizon h . We run this test for all our variables of the baseline. Figure 5 reports the results including the Ludvigson, Ma and Ng (2017) financial uncertainty measure (u01) as uncertainty proxy. If the zero line is not included in the confidence bands, then there will be evidence of state-dependent reactions.

According to Figure 5, we find statistically significant differences in the reactions of all variables included in our baseline specification across regimes.

5 Robustness checks

We check the robustness of our findings to a number of perturbations of the baseline STVAR model. In particular, we focus on i) different measures of uncertainty; ii) sample size and excluding the Zero Lower Bound period.

Alternative measures of financial uncertainty. In the vector (1), we implement the Ludvigson, Ma, and Ng (2017) measure with forecast horizon equal to 1-month (u01) as uncertainty proxy. As first robustness check, we estimate the STVAR using the uncertainty proxy relied on forecasting horizon equal to 3-months (u03) and to 12-months (u12). Figure 6 and Figure 7 plot the robust-

ness results for the recessionary and expansionary phases, respectively. The red, blue and green lines plotted the GIRFs when uncertainty proxied by the Ludvigson, Ma and Ng (2015) measures $u01$, $u03$ and $u12$, respectively. Figure 5 and Figure 6 show that qualitatively our baseline results are not affected by the horizon change. However, the forecast horizon of uncertainty measures affects quantitatively the macroeconomic effects of such shocks. Those results are in line with Ludvigson, Ma, and Ng (2017). Indeed, they point out that when the forecast horizons of their measures increase, the macroeconomic effects of financial uncertainty increase as well. Because of that, the Fed reacts via a stronger unconventional reaction that is higher as the forecasting horizons of increases. We repeat our exercise replacing the financial uncertainty Ludvigson, Ma, and Ng (2017) measures with an alternative indicator of uncertainty shock, the VXO.⁴ The magenta lines of figure 6 and 7 refer to the GIRFs when uncertainty proxied by the VXO. The reaction of macroeconomic variables is short-lived and smaller than the ones found relying on the Jurado, Ludvigson, and Ng (2015) proxies. Jurado, Ludvigson, and Ng (2015) provide evidence that effects of uncertainty shocks might depend on the source of the shocks and on its duration. Moreover, they found that the estimated duration of a shock to the VXO is around 4 months, whereas the one of macroeconomic uncertainty is much more persistent than the VXO. Our results are in line with the Jurado, Ludvigson, and Ng (2015) prediction. Interestingly, the Federal Reserve reacts to a VXO shocks through unconventional monetary policies as shown by the behavior of the Balance Sheet.

Sample size and ZLB. The baseline STVAR model is estimated on the sample from 1960M7 to 2018M12. The results concerning the asymmetric effects of financial shocks conditional on the state of economy may be too heavily driven by the inclusion of the Great Recession period in our sample. We investigate

⁴The VXO is employed instead of the VIX, since the VIX is available from 1990. The VXO is from 1985M1, the standard deviation of stock market returns as in Bloom (2009). From 1986M1 the VXO is from the Chicago Board of Options Exchange (CBOE).

about this issue repeating our analysis on the sample spanning to the month just prior to the recession, 1960M7 to 2007M11, and excluding from vector X_t our proxy for the "unconventional" monetary policy. In other words, we allow the Fed to react to uncertainty shocks only via "conventional" monetary tools (lowering the interest rate). Figure 8 depicts the results. In particular, the red lines refers to the median generalised IRFs (GIRFs) in recessions when uncertainty proxied by the Ludvigson, Ma and Ng (2017) measure (u01). The uncertainty shocks trigger macroeconomic fluctuation even excluding the ZLB period. Of course, the reaction of the macroeconomic aggregates is weaker, whereas the ones of the FFR is stronger than our baseline specification. This results is in line with Evans, Fisher, Gourio, and Krane (2015). They find that uncertainty is important to study the FFR pattern in the pre-ZLB period. Comparing Figure 8 and Figure 2 an interesting picture emerges. The presence of the ZLB may magnify the effects of uncertainty shocks (Basu and Bundick, 2015). Relying on "unconventional" monetary policy when the ZLB binds, we find that the Fed offsets the negative macroeconomic fluctuations.

Overall our robustness checks confirm the nonlinearity of uncertainty shock effects and their impact on the "conventional" and "unconventional" monetary policy decision of the Federal Reserve.⁵

6 Counterfactual Experiments

Did the Fed's Balance Sheet policies have a material impact on the US economy when uncertainty shocks occur? We answer this question proposing a counterfactual experiment in which we "switch down" the coefficients of the Balance

⁵Our results are also robust to: different ordering; lag specifications; different values of parameters that govern the transition from one regime to another. Moreover, our findings are qualitatively robust to the alternative proxy of uncertainty, such as the quarterly Rossi and Sekhposyan (2015) macroeconomic measure.

Sheet in the baseline VAR. In practical terms, we allow the Federal Reserve to react to a financial uncertainty shock via only the "conventional" monetary policy tools. The main findings provide evidence that the monetary policy responses rely on the Balance Sheet only during recessions. Hence, our counterfactual experiments are focused on these periods.

Figure 9 represents the reactions of macroeconomic activity to financial uncertainty shock in the counterfactual (dotted lines) and baseline scenarios (solid lines). Two results stand out. First of all, the macroeconomic effects of uncertainty become in absolute value larger than the ones derived from our baseline. Second, when the FED does not rely on the Balance Sheet increase the effects become more persistent. After 60 months, the macroeconomic variables are still below their pre-shock levels. It means that the "unconventional" monetary policy reaction is a powerful tool to counteract economic downturns due to uncertainty shock during recessionary periods.

In addition, we replicate the experiment introducing a new monetary policy tool: the Shadow Short Rate (SSR). We adopt the Shadow Short Rate introduced by Lombardi and Zhu (2014) which is an overall stance of the "conventional" and "unconventional" monetary policy. In this case, the counterfactual experiment assumes a SSR equals to zero. In the new baseline, we observe a negative short term interest rate since the SSR could assume negative values which are not allowed to the FFR.

Figure 10 reports the counterfactual experiment including the SSR instead of the short term interest rate (FFR) and the Balance Sheet (BS). Meanwhile, Figure 12 reports the experiment adding a financial variable such as the S&P500. As discussed in Bloom (2008) and Jurado, Ludvigson, and Ng (2015), the S&P500 indicator is crucial to explain the channel of the uncertainty shock on macroeconomic variables since the S&P500 is highly correlated to the uncertainty proxies

implemented in their researches. In the baseline specification, we exclude the S&P500 since it is not highly correlated with the financial uncertainty measure we adopt and using a nonlinear estimation for our model we keep the baseline model as parsimonious as possible. In Figure 11 and Figure 12, we note how the reaction of macroeconomic variables (Industrial Production and Employment) worsened with respect to the baseline specification. However, as expected the result for Inflation is different. There is a less persistent response in the case of counterfactual since the SSR is anchored to be zero and the more persistent response in the new baseline is driven by the negative SSR.

7 Conclusion

We estimate a nonlinear VAR model, the Smooth Transition VAR (STVAR), where we include standard macroeconomic variables and uncertainty proxies for the U.S. economy. We investigate the impact of the uncertainty shock on the monetary policies. For this purpose, we introduce both the "conventional" (short term interest rate) and "unconventional" (Balance Sheet) tools implemented by the Federal Reserve. The non-linearities inducted by the STVAR allow us to disentangle the behavior of the macroeconomic variables in two periods: recessions and expansions. Uncertainty shock is found to trigger negative macroeconomic fluctuations across the business cycle. To offset macroeconomic fluctuation, the Federal Reserve reacts lowering the FFR. However, when the FFR is close to zero uncertainty shocks push the Fed to react via non-standard monetary policy tools. Counterfactual experiments provide evidence about the role of the Balance Sheet during the recessionary periods.

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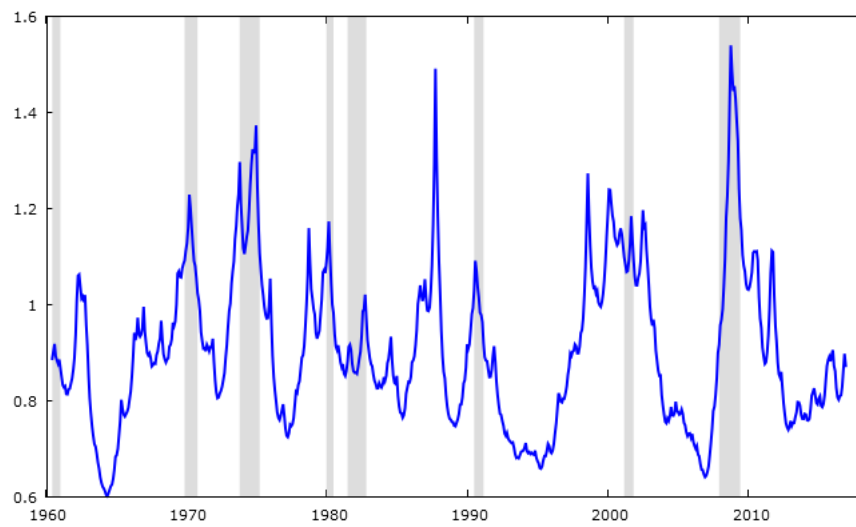
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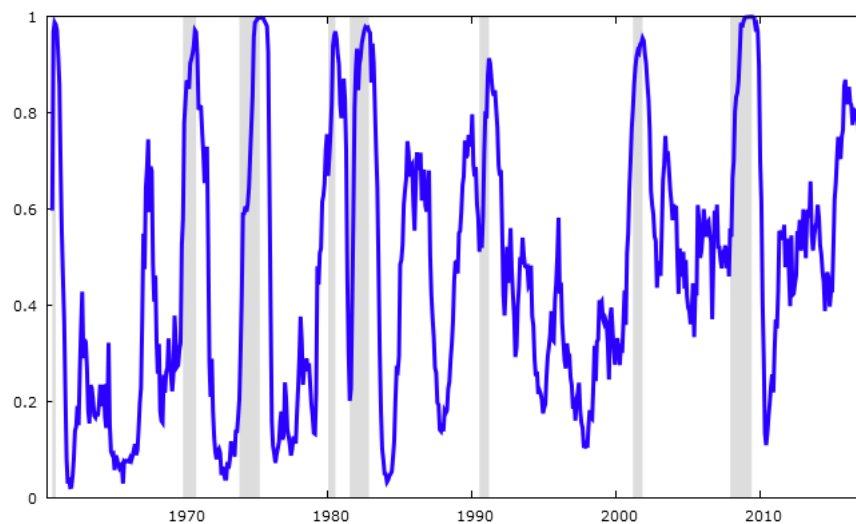
Figures

Figure 1: Uncertainty measures vs Business cycle



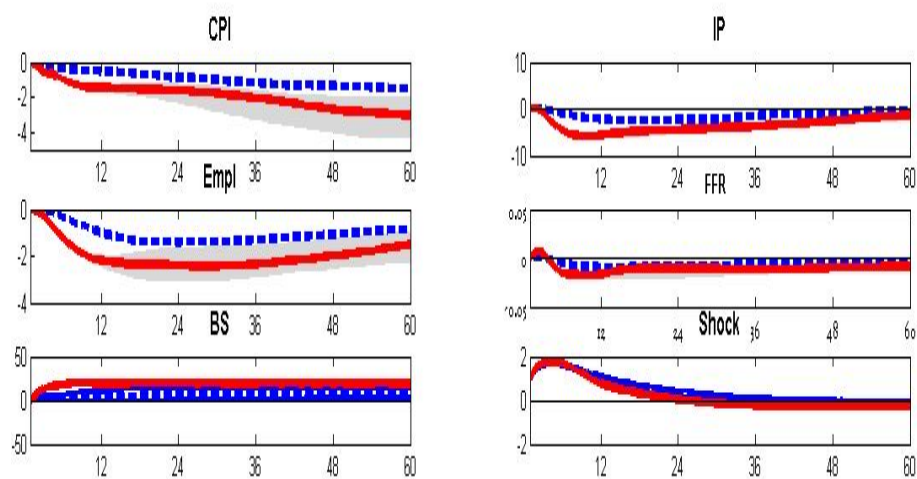
Notes: The shaded area indicate the U.S. recessionary phases (1960:7-2018:12), whereas the blue line refers to the uncertainty measure at 1 month proposed by Ludvigson, Ma and Ng (2017).

Figure 2: Transition function vs Business cycle



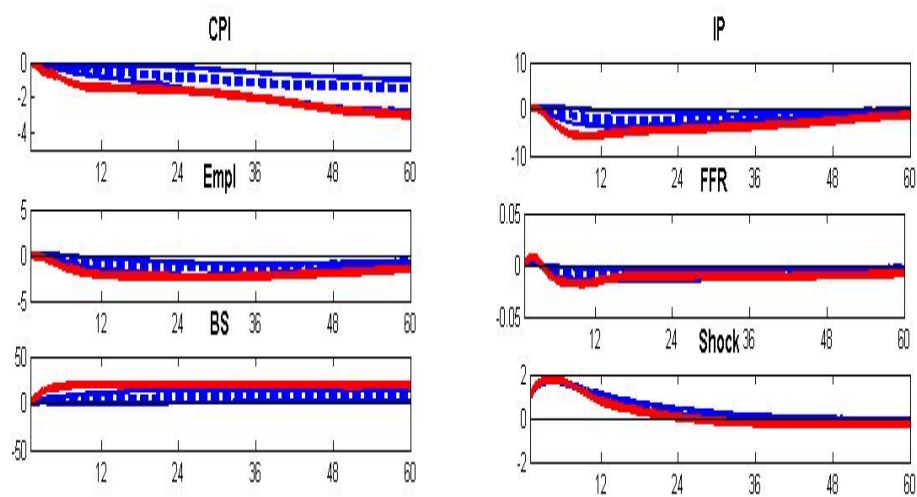
Notes: The shaded area indicate the U.S. recessionary phases (1960:7-2018:12), whereas the blue line refers to the backward looking 12-month moving average of IP growth.

Figure 3: Effects of uncertainty shocks



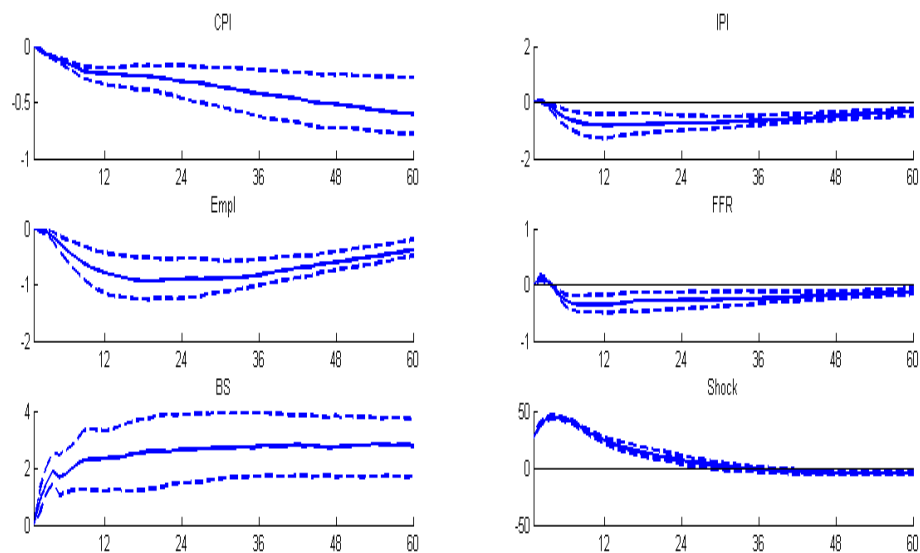
Notes: The red lines refers to the generalised IRFs (median) in recessions, whereas the blue lines to the ones in expansions. Uncertainty proxied by the Ludvigson, Ma and Ng (2017) measure (u01). Gray areas refers to the 68% confidence bands. The variables are expressed in percent deviations with respect to their steady state. The horizontal axis identifies months.

Figure 4: Effects of uncertainty shocks



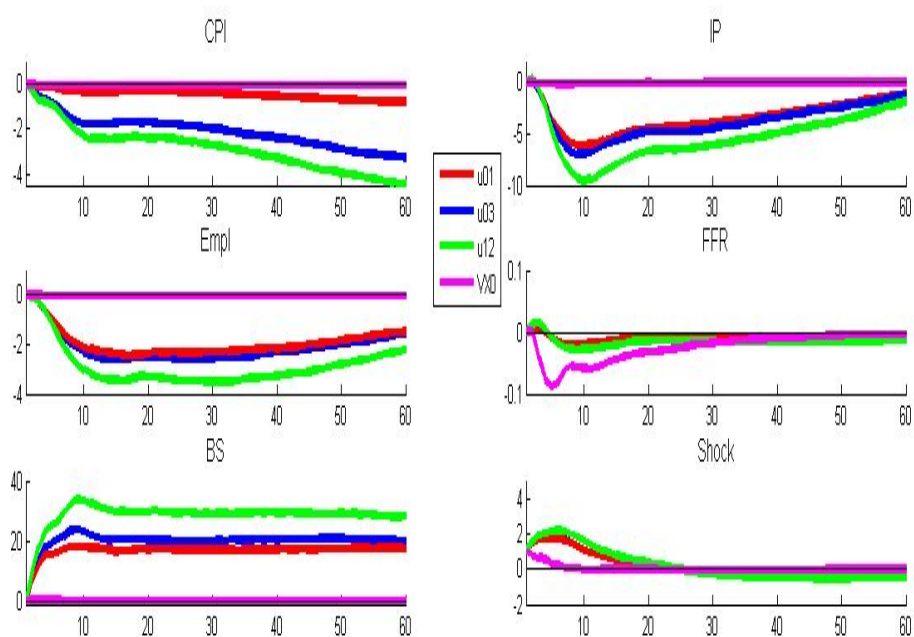
Notes: The red lines refers to the generalised IRFs (median) in recessions, whereas the blue lines to the ones in expansions. Uncertainty proxied by the Ludvigson, Ma and Ng (2017) measure (u01). Gray areas refers to the 68% confidence bands. The variables are expressed in percent deviations with respect to their steady state. The horizontal axis identifies months.

Figure 5: Differences in Generalized Impulse Responses between recessions and expansions (u01)



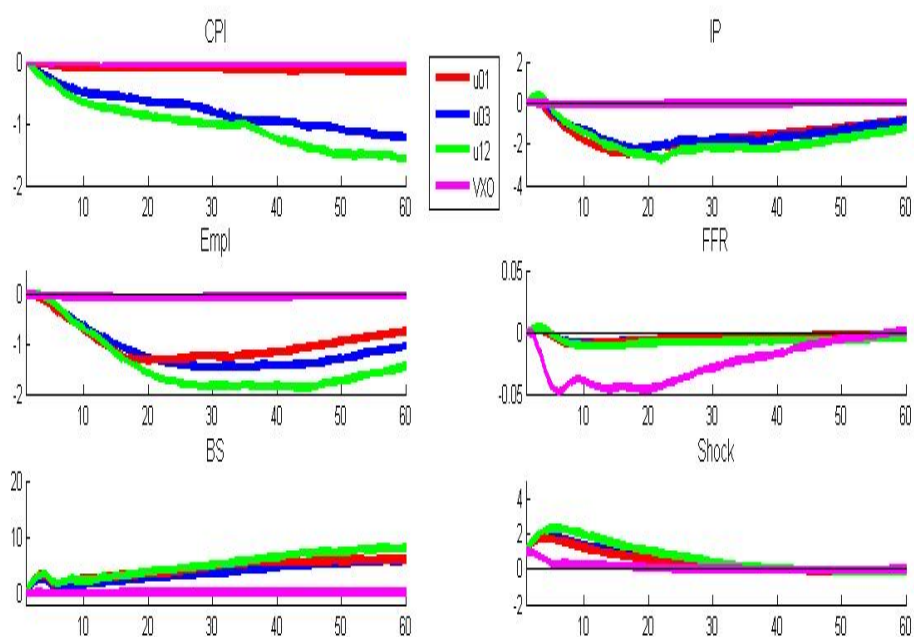
Notes: Differences in Generalized Impulse Responses between the recessions and expansions. Uncertainty proxied by the Ludvigson, Ma and Ng (2017) measure (u01). Dotted lines refer to the the 68% confidence bands. Horizontal axis denotes monthly horizon.

Figure 6: Effects of uncertainty shocks



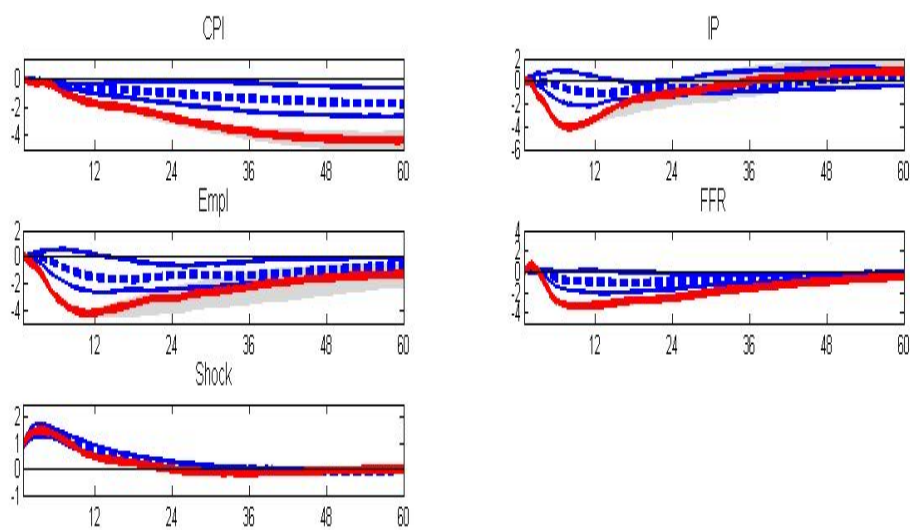
Notes: The figure plots the GIRFs to different uncertainty shocks occurring during recessionary periods. The red, blue and green lines plotted the GIRFs when uncertainty proxied by the Ludvigson, Ma and Ng (2015) measures u_{01} , u_{03} and u_{12} , respectively. The size spans from 1960M7 to 2018M12. The variables are expressed in percent deviations with respect to their steady state. The horizontal axis identifies months.

Figure 7: Effects of uncertainty shocks



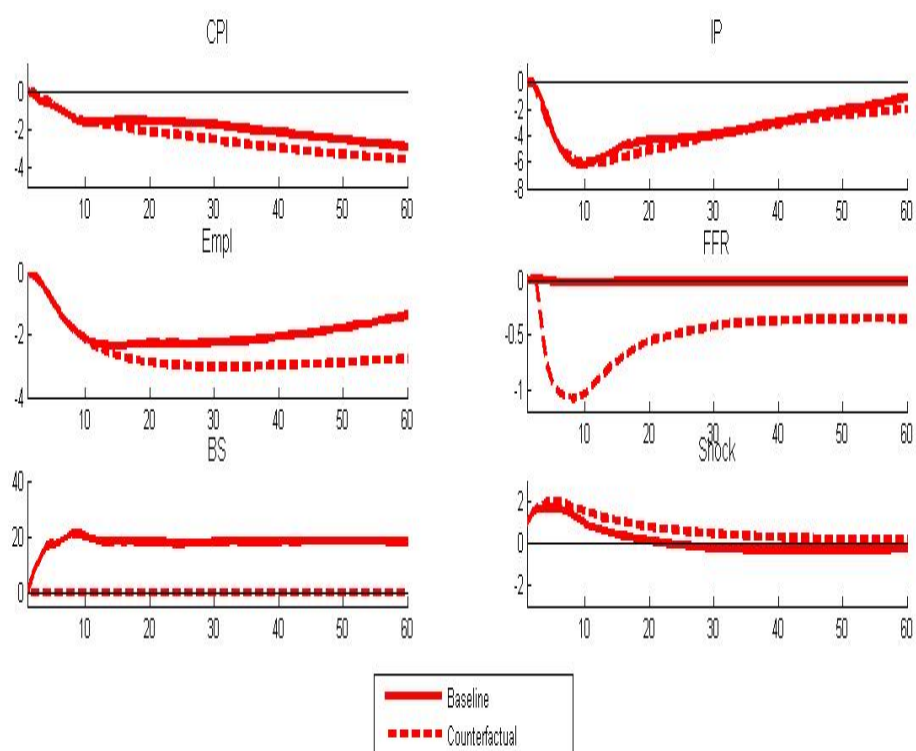
Notes: The figure plots the GIRFs to different uncertainty shocks occurring during expansionary periods. The red, blue and green lines plotted the GIRFs when uncertainty proxied by the Ludvigson, Ma and Ng (2015) measures u_{01} , u_{03} and u_{12} , respectively. The size spans from 1960M7 to 2018M12. The variables are expressed in percent deviations with respect to their steady state. The horizontal axis identifies months.

Figure 8: Effects of uncertainty shocks in recessions (from 1960M7 to 2007M11)



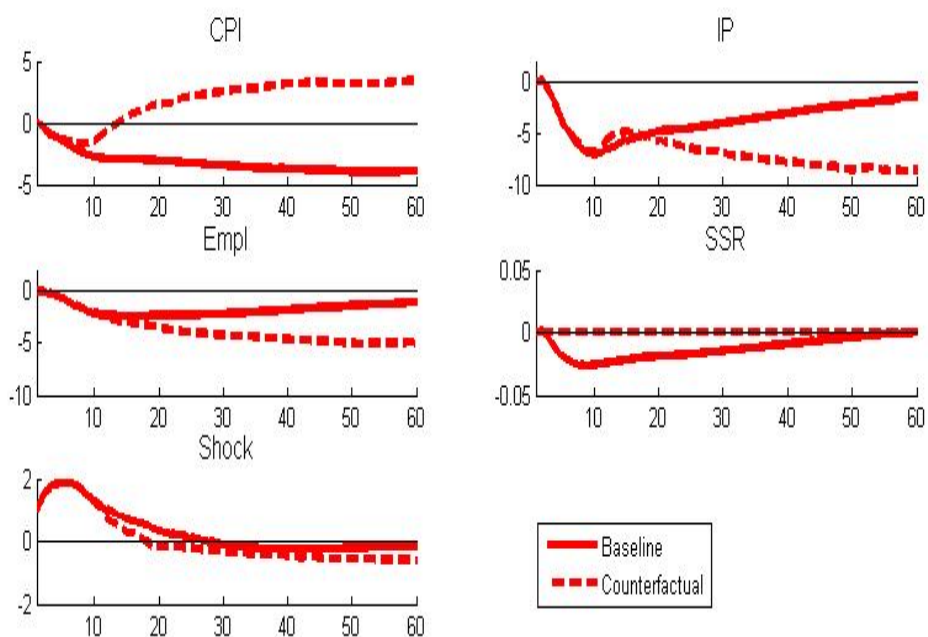
Notes: The red lines refers to the median generalised IRFs (GIRFs) in recessions when uncertainty proxied by the Jurado, Ludvigson and Ng (2015) measure ($u01$). The blue and green lines plotted the GIRFs when uncertainty proxied by the Ludvigson, Ma and Ng (2017) measures $u01$. The size spans from 1960M7 to 2007M11. The vector X_t includes only the proxy for the conventional monetary policy excluding the unconventional one. The variables are expressed in percent deviations with respect to their steady state. The horizontal axis identifies months.

Figure 9: GIRFs: Baseline vs Counterfactual



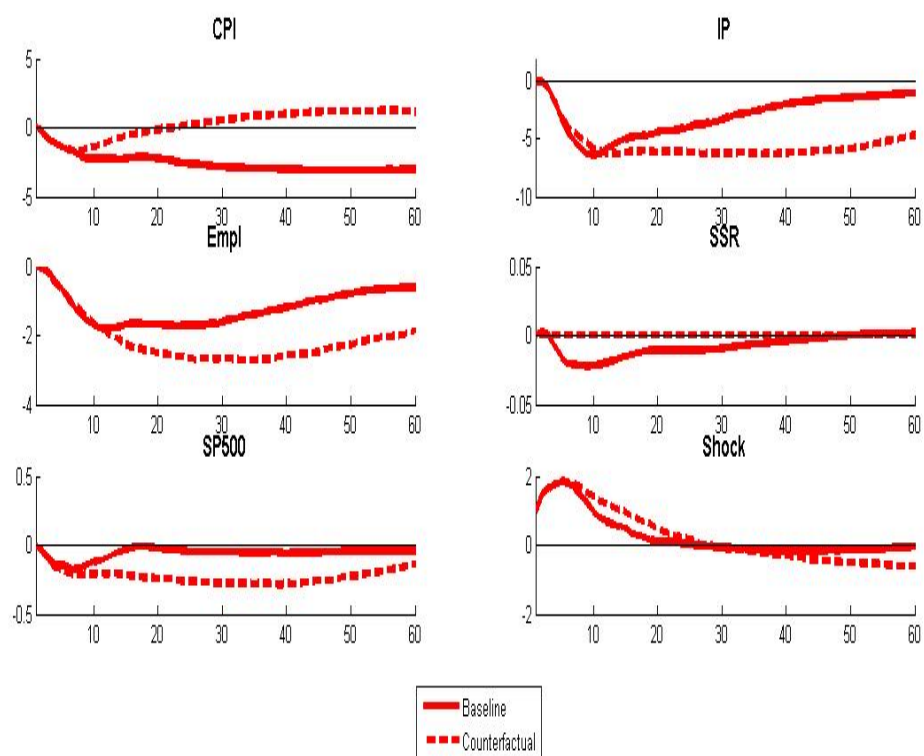
Notes: GIRFs in recessions. The solid and dotted red lines refer to the median generalised IRFs (GIRFs) from the baseline and the counterfactual scenarios. Uncertainty proxied by the Ludvigson, Ma, and Ng (2017) measure ($u01$). The variables are expressed in percent deviations with respect to their steady state. The horizontal axis identifies months. Horizontal axis denotes monthly horizon.

Figure 10: GIRFs: Baseline vs Counterfactual (with SSR)



Notes: GIRFs in recessions. The solid and dotted red lines refer to the median generalised IRFs (GIRFs) from the baseline and the counterfactual scenarios. Ludvigson, Ma, and Ng (2017) measure (u01). The conventional and unconventional monetary policy indicators are replaced by the shadow short rate (SSR) proxy proposed by Lombardi and Zhu (2014). The variables are expressed in percent deviations with respect to their steady state. The horizontal axis identifies months. Horizontal axis denotes monthly horizon.

Figure 11: GIRFs: Baseline vs Counterfactual (with SSR and S&P500)



Notes: GIRFs in recessions. The solid and dotted red lines refer to the median generalised IRFs (GIRFs) from the baseline and the counterfactual scenarios. udvingson, Ma, and Ng (2017) measure (u01). The conventional and unconventional monetary policy indicators are replaced by the shadow short rate proxy proposed by Lombardi and Zhu (2014). Moreover, the S&P500 is added to the vector of endogenous variables. The variables are expressed in percent deviations with respect to their steady state. The horizontal axis identifies months. Horizontal axis denotes monthly horizon.

Technical Appendix

This Technical Appendix reports the estimation of the non-linear VARs, the statistical evidence in favor of a nonlinear relationship between the endogenous variables included in the STVAR, and the computation of the Generalised Impulse Responses. All these Sections are partially drawn on Caggiano, Castelnuovo, Colombo, and Nodari (2015) Appendix.

A Linearity Test

We test linearity versus non-linearity applying the Teräsvirta and Yang (2014) test for Smooth Transition Vector AutoRegression (STVAR) with a single transition variable as in our framework. According to this test, we assume linearity under null hypothesis versus a nonlinear model with a logistic smooth transition component under alternative hypothesis. Let us assume a p -dimensional 2-regime approximate logistic STVAR model:

$$\mathbf{X}_t = \Theta_o' Y_t + \sum_{i=1}^n \Theta_i' \mathbf{Y}_t z_t^i + \varepsilon_t, \quad (5)$$

where \mathbf{X}_t is the ($p \times 1$) vector of endogenous variables, $\mathbf{Y}_t = [\mathbf{X}_{t-1} \mid \dots \mid \mathbf{X}_{t-k}]$ is the ($k \times p+q$) vector of exogenous variables which includes lagged variables (k) and a vector of constants. The transition variable is z_t , while Θ_o and Θ_i are matrices of parameters. In our empirical assessment, we have $p=9$ as number of endogenous variables, $q=1$ as number of exogenous variables, and $k=5$ as number of lags. Under the null hypothesis of linearity, we assume $\mathbf{H}_o : \Theta_i=0 \forall i$. The Teräsvirta and Yang (2014) test features the following four steps:

1) We estimate the restricted model ($\mathbf{H}_o : \Theta_i=0 \forall i$) by regressing \mathbf{X}_t on \mathbf{Y}_t . We collect the residual $\tilde{\mathbf{E}}$ calculating the matrix for the residual sum of squares $RSS_0 = \tilde{\mathbf{E}}' \tilde{\mathbf{E}}$.

2) We run an auxiliary regression of $\tilde{\mathbf{E}}$ on $(\mathbf{Y}_t, \mathbf{Z}_n)$ where the subscript n indicates the n -order Taylor expansion of the transition function. We save the residuals $\tilde{\mathbf{E}}$ computing the matrix for the residual sum of squares $RSS_1 = \tilde{\mathbf{E}}' \tilde{\mathbf{E}}$.

3) We compute the test-statistic:

$$LM = Ttr[RSS_0^{-1}(RSS_0 - RSS_1)] = T[p - tr(RSS_0^{-1}RSS_1)]. \quad (6)$$

Under the null hypothesis, the test statistic is distributed as a χ^2 with a number of degree of freedoms equals the number of restrictions, $p(kp+q)$. We compute two LM-type linearity tests fixing the value of the n -order of the Taylor expansion equal to $n = 1$ and $n = 3$ (as proposed by Luukkonen, Saikkonen, and Teräsvirta, 1988). In our estimation, LM=791 and LM=1738 when $n = 1$ and $n = 3$, respectively. The corresponding p-value in both tests are zero. In other words, our model is present non-linear dynamics.

B Estimation of the Non-linear VARs

Our STVAR model (1)-(4) is estimated via maximum likelihood. The log - likelihood function is as follows:

$$\log L = const - \frac{1}{2} \sum_{t=1}^T \log |\Omega_t| - \frac{1}{2} \sum_{t=1}^T \varepsilon_t' \Omega^{-1} \varepsilon_t, \quad (7)$$

where the vector of residuals $\varepsilon_t = X_t - (1 - F(z_t))\Pi_E X_{t-1} - F(z_t)\Pi_R X_{t-1}$. Our purpose is to estimate the parameters $\Psi = \{\Omega_R, \Omega_E, \Pi_R(L), \Pi_E(L)\}$, where $\Pi_j(L) = [\Pi_{j,1}, \dots, \Pi_{j,p}]$, $j \in \{R, E\}$.

Due to the high non-linearity of the model its estimation is problematic using standard optimisation procedures. Hence, as in Auberbach and Gorodnichenko (2012), we employ the procedure as described as follows.

Conditional on $\gamma, \Omega_R, \Omega_E$, where γ is the slope parameter calibrated as described in section 2, the model is linear in Π_R, Π_E . Hence, for a given guess on $\gamma, \Omega_R, \Omega_E$, the coefficients Π_R, Π_E can be estimated by minimizing $\frac{1}{2} \sum_{t=1}^T \varepsilon'_t \Omega^{-1} \varepsilon_t$. Hence, we can re-write the regressors as below.

Let $\mathbf{W}_t = [F(z_t)\mathbf{X}_{t-1}(1-F(z_t))\mathbf{X}_{t-1} \dots F(z_t)\mathbf{X}_{t-p}(1-F(z_t))\mathbf{X}_{t-p}]$ be the extended vector of regressors, and $\mathbf{\Pi} = [\mathbf{\Pi}_R(L)\mathbf{\Pi}_E(L)]$. Consequently, we can write $\varepsilon_t = \mathbf{X}_t - \mathbf{\Pi}\mathbf{W}'_t$. In this case, the objective function becomes:

$$\frac{1}{2} \sum_{t=1}^T (\mathbf{X}_t - \mathbf{\Pi}\mathbf{W}'_t)' \Omega_t^{-1} (\mathbf{X}_t - \mathbf{\Pi}\mathbf{W}'_t). \quad (8)$$

We can show that the first order condition with respect to $\mathbf{\Pi}$ is given by:

$$vec \mathbf{\Pi}' = \left(\sum_{t=1}^T [\Omega_t^{-1} \otimes \mathbf{W}'_t \mathbf{W}_t] \right)^{-1} vec \left(\sum_{t=1}^T \mathbf{W}'_t \mathbf{X}_t \Omega_t^{-1} \right). \quad (9)$$

We iterate this procedure over different sets of values for $\{\Omega_R, \Omega_E\}$ (conditional on a given value for γ). For each set of values, $\mathbf{\Pi}$ is obtained and the $logL$ (7) is calculated.

Due to the high non-linearity of the model in its parameters, we might get several local optima. Then, it is recommended to try different starting values of γ . To guarantee positive definiteness of the matrices Ω_R and Ω_E , we focus on the alternative vector of parameters $\Psi = \{chol(\Omega_R), chol(\Omega_E), \mathbf{\Pi}_R(L), \mathbf{\Pi}_E(L)\}$, where *chol* means the Cholesky decomposition.

We compute the confidence intervals using a Markov Chain Monte Carlo (MCMC) algorithm developed by Chernozhukov and Hong (2003) (CH hereafter). This methodology gives us both a global optimum and densities for the parameter estimates.

We implement the CH estimation via a Metropolis-Hastings algorithm. Given a starting value Ψ^0 , the procedure constructs chains of length N of the parameters

of the estimated model following two steps:

Step 1: Draw a candidate vector of parameter values $\Theta^{(n)} = \Psi^{(n)} + \psi^{(n)}$ for the chain's $n + 1$ state, where $\Psi^{(n)}$ is the current state and $\psi^{(n)}$ is a vector of i.i.d. shocks drawn from $N(0, \Omega_{\Psi})$, and Ω_{Ψ} is a diagonal matrix.

Step 2: Set the $n + 1$ state of the chain $\Psi^{(n+1)} = \Theta^{(n)}$ with probability $\min\{1, L(\Theta^{(n)})/L(\Psi^{(n)})\}$, where $L(\Theta^{(n)})$ is the value of the likelihood function conditional on the candidate vector of parameter values, and $L(\Psi^{(n)})$ is the value of the likelihood function conditional on the current state of the chain. Otherwise, set $\Psi^{(n+1)} = \Psi^{(n)}$.

The starting value $\Theta^{(0)}$ is calculated using the second-order Taylor approximation of the model described from (1) to (4) in the section 2, hence the model can be written as regressing \mathbf{X}_t , $\mathbf{X}_t z_t$, and $\mathbf{X}_t z_t^2$. We employ the residuals from this regression to fit the expression for the reduced-form time-varying variance-covariance matrix of the VAR (as explained in the main text) using maximum likelihood to estimate Ω_R and Ω_E .

We can construct Ω_t , conditional on these estimates and given the calibration for γ . Conditional on Ω_t , we can compute the starting values for $\Pi_R(L)$ and $\Pi_E(L)$ using equation (9).

Given the calibration for the initial (diagonal matrix) Ω_{Ψ} , a scale factor is adjusted to generate an acceptance rate close to 0.3, the typical value for this computational methods as pointed out by Canova (2007). The estimation accounts for $N = 50,000$ draws and we use the last 20% for inference.

As described by CH, $\Psi^* = \frac{1}{N} \sum_{t=1}^T \Psi^{(n)}$ is consistent estimate of Ψ under standard regularity assumptions on maximum likelihood estimators. The covariance matrix of Ψ is given by $V = \frac{1}{N} \sum_{t=1}^T (\Psi^{(n)} - \Psi^*)^2 = var(\Psi^{(n)})$, which is the variance of the estimates in the generated chain.

C Generalized Impulse Response Functions

The Impulse Response Functions for the STVAR model are computed following the approach introduced by Koop, Pesaran, and Potter (1996) which propose an algorithm to calculate the Generalized Impulse Response Functions (GIRFs). The implementation of their procedure is composed of the following steps.

- 1) We construct the set of all possible histories $\mathbf{\Lambda}$ of length $p = 12 : \{\lambda_i \in \mathbf{\Lambda}\}$, where $\mathbf{\Lambda}$ contain $T - p + 1$ histories λ_i and T is the sample size ($T=312$).
- 2) We separate the set of all recessionary histories from that of all expansionary histories. We calculate the transition variable z_{λ_i} for each λ_i . If $z_{\lambda_i} \leq z^* = -0.9\%$, then $\lambda_i \in \mathbf{\Lambda}^R$, where $\mathbf{\Lambda}^R$ refers to all recessionary histories; if $z_{\lambda_i} > z^* = -0.9\%$, then $\lambda_i \in \mathbf{\Lambda}^E$, where $\mathbf{\Lambda}^E$ refers to all expansionary histories.
- 3) We select at random one history λ_i from the set $\mathbf{\Lambda}^R$, taking $\hat{\mathbf{\Omega}}_{\lambda_i}$ obtained as follows:

$$\hat{\mathbf{\Omega}}_{\lambda_i} = F(z_{\lambda_i})\hat{\mathbf{\Omega}}_R + (1 - F(z_{\lambda_i}))\hat{\mathbf{\Omega}}_E, \quad (10)$$

where z_{λ_i} is the transition variable computed for the selected history λ_i . $\hat{\mathbf{\Omega}}_R$ and $\hat{\mathbf{\Omega}}_E$ are calculated from the generated MCMC chain of the parameter values during the estimation step. As in Koop et al. (1996), we consider the distribution of parameters rather than their mean values to allow for parameter uncertainty.

- 4) We estimate the variance-covariance matrix $\hat{\mathbf{\Omega}}_{\lambda_i}$ using the Cholesky-decomposition:

$$\hat{\mathbf{\Omega}}_{\lambda_i} = \hat{\mathbf{C}}_{\lambda_i} \hat{\mathbf{C}}'_{\lambda_i}, \quad (11)$$

we orthogonalize the estimated residuals to get the structural shocks as:

$$\mathbf{e}_{\lambda_i}^{(j)} = \hat{\mathbf{C}}_{\lambda_i}^{-1} \hat{\varepsilon}. \quad (12)$$

5) From \mathbf{e}_{λ_i} draw with replacement h nine-dimensional shocks and get the vector of bootstrapped shocks

$$\mathbf{e}_{\lambda_i}^{(j)*} = \{\mathbf{e}_{\lambda_i,t}^*, \mathbf{e}_{\lambda_i,t+1}^*, \dots, \mathbf{e}_{\lambda_i,t+h}^*\}, \quad (13)$$

where h is the number of horizons for the IRFs we compute.

6) We form another set of bootstrapped shocks which are equal to (13) except for the k_{th} shock in $\mathbf{e}_{\lambda_i}^{(j)*}$ which is the shock we perturb by a δ amount. We call the vector of bootstrapped perturbed shocks as $\mathbf{e}_{\lambda_i}^{(j)\delta}$.

7) We transform back $\mathbf{e}_{\lambda_i}^{(j)*}$ and $\mathbf{e}_{\lambda_i}^{(j)\delta}$ as follows:

$$\hat{\varepsilon}_{\lambda_i}^{(j)*} = \hat{\mathbf{C}}_{\lambda_i} \mathbf{e}_{\lambda_i}^{(j)*}, \quad (14)$$

and

$$\hat{\varepsilon}_{\lambda_i}^{(j)\delta} = \hat{\mathbf{C}}_{\lambda_i} \mathbf{e}_{\lambda_i}^{(j)\delta}. \quad (15)$$

8) We use (14) and (15) to simulate the evolution of $\mathbf{X}_{\lambda_i}^{(j)*}$ and $\mathbf{X}_{\lambda_i}^{(j)\delta}$ and we construct the $GIRF^{(j)}(h, \delta, \lambda_i)$ as $\mathbf{X}_{\lambda_i}^{(j)*} - \mathbf{X}_{\lambda_i}^{(j)\delta}$.

9) Conditional on history λ_i , repeat for $j=1, \dots, B$ vectors of bootstrapped residuals and get $GIRF^1(h, \delta, \lambda_i)$, $GIRF^2(h, \delta, \lambda_i)$, ..., $GIRF^B(h, \delta, \lambda_i)$. We set $B=500$.

10) We calculate the GIRF conditional on history λ_i as:

$$\hat{GIRF}^{(i)}(h, \delta, \lambda_i) = B^{-1} \sum_{j=1}^B GIRF^{(i,j)}(h, \delta, \lambda_i). \quad (16)$$

11) We repeat all previous steps for $i=1, \dots, 500$ histories belonging to the set of recessionary histories, $\lambda_i \in \Lambda^R$, and we get $\hat{GIRF}^{(1,R)}(h, \delta, \lambda_{1,R})$, $\hat{GIRF}^{(2,R)}(h, \delta, \lambda_{2,R})$, ..., $\hat{GIRF}^{(500,R)}(h, \delta, \lambda_{500,R})$ where the subscript R means that we are condition-

ing upon recessionary histories.

12) We take the average and we get $G\hat{I}RF^{(R)}(h, \delta, \Lambda_R)$, which is the average GIRF under recessions.

13) We repeat all the previous steps from 3 to 12 for 500 histories belonging to the set of all expansions and we get $G\hat{I}RF^{(E)}(h, \delta, \Lambda_E)$.

14) We compute the 68% confidence bands for the IR by picking up for each horizon of each state, the 16th and 84th percentile of the densities $G\hat{I}RF^{([1:500],R)}$ and $G\hat{I}RF^{([1:500],E)}$.