MOMENT SEQUENCES AND ORTHOGONAL POLYNOMIALS

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After research in potential theory I have for the last 40 years been fascinated by moment problems and orthogonal polynomials, an area with an interplay between complex analysis, functional analysis and operator theory. Although it is a very classical subject, there has been a lot of recent work, see e.g. [1]. My own research has been focused on what is called indeterminate moment problems, i.e. the case where different positive measures have the same moments.

I will define moment sequences of Stieltjes, Hamburger and Hausdorff and explain how they are characterized. I will also tell a little about the fascinating life and work of the Dutch mathematician Stieltjes, and why he ended up in Toulouse.

I will discuss the theory of orthogonal polynomials associated with a positive measure on the real line and give examples of determinate and indeterminate moment problems.

I will also discuss a joint result with Antonio Durán from Seville about a "product transformation" from Hausdorff moment sequences to Stieltjes moment sequences, showing that $(n!)^c$ is a Stieltjes moment sequence for any $c > 0$. This result was inspired by work about exponential functionals of Levy processes due to the French School of probabilists around Bertoin and Yor. It turns out that $(n!)^c$ is determinate for $0 < c \leq 2$ but indeterminate when $c > 2$.

I shall end with some new results of Yafaev, Szwarc and myself about closability of certain operators related to moment problems.

References

[1] M. E. H. Ismail, *Classical and Quantum Orthogonal Polynomials in One Variable*, Cambridge University Press, Cambridge 2005.